From Featured Transition Systems to Modal Transition Systems (with variability constraints) and a Variability Model Checker

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Modelling and Analysis of Variability in Software Product Lines
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Outline

1. Background: Software Product Line Engineering

2. Formal Methods and Analysis in Software Product Line Engineering
   - Recall: Featured Transition Systems
   - Modal Transition Systems with variability constraints
   - Transformation: from feature constraints to action constraints

3. v-ACTL: variability-aware, action-based CTL
   - Preservation of formulae in v-ACTL \( \Box \)/v-ACTL\( \Diamond \)

4. VMC: Variability Model Checker
   - Family-based and product-based verification with VMC
   - A value-passing modal process algebra
   - An example family of Bike-Sharing Systems (BSS)
Formal methods and tools in SPLE

Computer-aided analysis of feature models
- Traditionally: focus on modelling/analysing structural constraints
- But: software systems often embedded/distributed/safety-critical
- Important: model/analyse also behaviour (e.g. quality assurance)

Goal: rigorously establish critical requirements of (software) systems
⇒ lift success stories from single product/system engineering to SPLE

Widely used behavioural SPL models with dedicated model checkers
- Modal Transition Systems (MTS) with variability constraints
  Fantechi, Gnesi @ SPLC’08, Asirelli et al. @ iFM’10, SPLC’11, ter Beek et al. @ JLAMP, 2016
  Variability Model Checker VMC
  ter Beek et al. @ FM’12, SPLC’12, SPLat’14
- Featured Transition Systems (FTS)
  SNIP, ProVeLines, NuSMV extension
  Classen et al. @ ICSE’11, Int. J. Softw. Tools Technol. Transf., 2012, Cordy et al. @ SPLC’13
Recall (atomic propositions used for verification purposes):
De Nicola, Vaandrager @ J. ACM, 1995

A **Doubly-Labelled Transition System** ($L^2TS$) is a sextuple $(Q, A, \overline{q}, \rightarrow, AP, L)$ with states $Q$, actions $A$, initial state $\overline{q}$, transitions $\rightarrow \subseteq Q \times A \times Q$, atomic propositions $AP$, and labeling function $L : S \rightarrow 2^{AP}$

An FTS adds to this a feature model and feature expressions:

A **Featured Transition System** (FTS) is an octuple $(Q, A, \overline{q}, \rightarrow, AP, L, FD, \gamma)$ with underlying $L^2TS$ $(Q, A, \overline{q}, \rightarrow, AP, L)$, feature diagram $FD$ over a set $F$ of features, and total function $\gamma : \rightarrow \rightarrow \mathbb{B}(F)$ labelling each transition with a feature expression, i.e. a Boolean expression over the features
FTS of example SPL: a vending machine

Feature model:

12 valid products e.g. \{v, b, s, t\}, \{v, b, s, c\}
FTS of example SPL: a vending machine

Feature model:

FTS of 12 valid products (LTS) e.g. \{v, b, s, t\}, \{v, b, s, c\}
FTS of example SPL: a vending machine

Feature model:

- **VendingMachine**
  - **Beverages**
    - **Soda**
    - **Tea**
  - **FreeDrinks**
  - **CancelPurchase**

Example feature sets:
- `{v, b, s, t}`
- `{v, b, s, c}`

Workflow:
1. **Pay**
2. **Change**
3. **Serve Soda**
4. **Serve Tea**
5. **Serve Soda**
6. **Serve Tea**
7. **Open**
8. **Take**
9. **Close**
FTS of example SPL: a vending machine

Feature model:

VendingMachine

Beverages

FreeDrinks

CancelPurchase

Soda

Tea

and

or

optional

e.g. \{v, b, s, t\}, \{v, b, s, c\}
Main ingredient: Modal Transition Systems (MTS)

- LTS distinguishing possible (may) and required (must) transitions
  Larsen, Thomsen @ LICS'88

- Recognized as a useful model to describe in a compact way the possible behaviour of all the products (LTS) of a product family
  Fischbein, Uchitel, Braberman @ ROSATEA'06, Fantechi, Gnesi @ ESEC/FSE'07, SPLC’08

- MTS cannot model variability constraints regarding alternative features, nor regarding requires/excludes inter-feature relations, resulting in several variants and extensions
  Larsen et al. @ ESOP’07, Lauenroth et al. @ ASE’09

- Our solution: add a set of variability constraints to the MTS to be able to decide which derivable products (LTS) are valid ones
  ter Beek, Fantechi, Gnesi, Mazzanti @ JLAMP, 2016
Recall:

A **Labelled Transition System** (LTS) is a quadruple $(Q, A, \overline{q}, \rightarrow)$ with states $Q$, actions $A$, initial state $\overline{q}$ and transitions $\rightarrow \subseteq Q \times A \times Q$

Next we define MTS with variability constraints:

A **Modal Transition System** (MTS) is a quintuple $(Q, A, \overline{q}, \rightarrow^\square, \rightarrow^\lozenge)$ such that $(Q, A, \overline{q}, \rightarrow^\square \cup \rightarrow^\lozenge)$ is an LTS, called its underlying LTS

An MTS has two distinct transition relations

1. **may** transition relation $\rightarrow^\lozenge \subseteq Q \times A \times Q$: possible transitions
2. **must** transition relation $\rightarrow^\square \subseteq Q \times A \times Q$: required transitions

By definition, any required transition is also possible: $\rightarrow^\square \subseteq \rightarrow^\lozenge$

(denote $\rightarrow^\square \equiv \rightarrow^\lozenge \setminus \rightarrow^\square$: optional transitions)
Variability constraints of form **ALTernative**, **EXCludes**, **REQuires**, etc.

- **a₁ ALT⋯ALT aₙ**: precisely one among the \( n \geq 2 \) actions \( a₁, \ldots, aₙ \) is reachable in \( L \) (i.e. is the label of a reachable transition)
- **b₁ OR⋯OR bₙ**, where \( bᵢ \) is either \( aᵢ \) or \( ¬aᵢ \): at least one among the conditions on \( n \geq 2 \) actions \( b₁, \ldots, bₙ \) holds, i.e. \( bᵢ = aᵢ \) is reachable in \( L \) or \( bᵢ = ¬aᵢ \) is not reachable in \( L \)
- **a₁ EXC a₂**: at most one of the actions \( a₁ \) and \( a₂ \) is reachable in \( L \)
- **a₁ REQ a₂**: action \( a₂ \) is reachable in \( L \) whenever \( a₁ \) is reachable in \( L \)
- **a₁ REQ (a₂ ALT⋯ALT aₙ)**: precisely one among the \( n \geq 2 \) actions \( a₂, \ldots, aₙ \) is reachable in \( L \) if \( a₁ \) is reachable in \( L \)
- **a₁ REQ (a₂ OR⋯OR aₙ)**: at least one among the \( n \geq 2 \) actions \( a₂, \ldots, aₙ \) is reachable in \( L \) if \( a₁ \) is reachable in \( L \)
Derive products (implemented in VMC)

A product LTS is obtained from a family MTS in the following way:

1. include all (reachable) must transitions and
2. include subset of the (reachable) optional transitions, remove rest
3. satisfy assumptions of coherence and consistency
4. satisfy variability constraints

⇒ Each selection gives rise to a different variant

Let \((Q, A, \bar{q}, \delta^{\Diamond}, \delta^{\Box})\) be a coherent MTS, i.e. \(\exists \overset{a}{\longrightarrow} \Rightarrow \not\exists \overset{a}{\longrightarrow}\)

The set \(\{P_i = (Q_i, A, \bar{q}, \delta_i) \mid i > 0\}\) of product LTS is obtained by considering each pair of \(Q_i \subseteq Q\) and \(\delta_i \subseteq \delta^{\Diamond} \cup \delta^{\Box}\) to be defined s.t.

1. every \(q \in Q_i\) is reachable in \(P_i\) from \(\bar{q}\) via transitions from \(\delta_i\)
2. there exists no \((q, a, q') \in \delta^{\Box} \setminus \delta_i\) such that \(q \in Q_i\)
3. LTS is consistent: both \(\overset{a}{\longrightarrow} \sim \overset{a}{\longrightarrow}\) and \(\not\overset{a}{\longrightarrow}\) not allowed
Derive products (implemented in VMC)

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$\Rightarrow$ Each selection gives rise to a different variant

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The set $\{ P_i = (Q_i, A, \bar{q}, \delta_i) \mid i > 0 \}$ of product LTS is obtained by considering each pair of $Q_i \subseteq Q$ and $\delta_i \subseteq \delta^\Box \cup \delta^\square$ to be defined s.t.

1. every $q \in Q_i$ is reachable in $P_i$ from $\bar{q}$ via transitions from $\delta_i$
2. there exists no $(q, a, q') \in \delta^\square \setminus \delta_i$ such that $q \in Q_i$
3. LTS is consistent: both $\xrightarrow{a} \not\xrightarrow{a}$ and $\not\xrightarrow{a}$ not allowed
Input: specification of an MTS in process-algebraic terms, together with a set of variability constraints

VMC offers two kinds of behavioural variability analyses (more later):

1. The actual set of all valid product behaviour can explicitly be generated and the resulting LTS can all be verified against one and the same logic property (expressed in Action-based CTL)

De Nicola, Vaandrager @ J. ACM, 1995

2. A logic property (expressed in variability-aware ACTL, more later) can directly be verified against the MTS, relying on the fact that under certain syntactic conditions validity over the MTS implies validity of the same property for all its products

ter Beek, Fantechi, Gnesi, Mazzanti @ JLAMP, 2016

VMC (v6.3, released in November 2015) is freely usable online:

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Crux: from feature constraints to action constraints

From feature model: A requires C

From FTS to MTS (naive): a REQ c

From FTS to MTS (solution):
1. new action \( \forall \) feature (to handle more complex feature expressions)
2. dummy transition \( \forall \) action (to verify constraints, ignored when model checking)

Consistency guarantees that whenever a c-labelled may transition from the initial state is preserved in this LTS, then also any other reachable c-labelled may transition must be preserved.
Crux: from feature constraints to action constraints

From feature model: A requires C

FTS
\[ \begin{array}{c}
1 & \xrightarrow{a/A} & 2 & \xrightarrow{b/B} & 3 & \xrightarrow{c/C} & 4 \\
\end{array} \]

LTS
\[ \begin{array}{c}
1 & \xrightarrow{a} & 2 \\
\end{array} \]

valid product (namely \( A \land C \land \neg B \))

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valid product (violates a REQ c)

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\(\{a,b,c,A,B,C\}\) \(\xrightarrow{s}\)

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Consistency guarantees that whenever a \( c \)-labelled may transition from the initial state is preserved in this LTS, then also any other reachable \( c \)-labelled may transition must be preserved.
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- LTS: \( 1 \xrightarrow{a} 2 \)
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Model Transformation (1/4)

Step 1: definition of valid products in terms of features

Translate feature model in a set of variability constraints on features
Model Transformation (1/4)

Step 1: definition of valid products in terms of features

Translate feature model in a set of variability constraints on features

Constraints { }

Soda OR Tea

Graphical representation of a feature model with constraints:

- **VendingMachine**
  - **Beverages**
    - **Soda**
    - **Tea**
  - **FreeDrinks**
  - **CancelPurchase**

Constraints: Soda OR Tea
Step 2: definition of valid products in terms of actions

Define logic formula ‘a ↔ φ’ for each transition \( a/\phi \rightarrow \) in FTS (feature expressions not translatable in VMC format are transformed in CNF, which is the reason for which the \( n \)-ary OR construct \( b_1 \text{ OR } \cdots \text{ OR } b_n \) can now contain either \( b_i = a_i \) or its negation \( b_i = \neg a_i \)).

Constraints {
  free IFF FreeDrinks
  pay ALT FreeDrinks
  cancel IFF CancelPurchase
  soda IFF Soda
  tea IFF Tea
  takeFree IFF FreeDrinks
  open ALT FreeDrinks
}
Step 2: definition of valid products in terms of actions

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}
Step 3: definition of valid products in MTS and variability constraints

1. Define FTS process algebraically, using ‘a(may)’ for each ‘a/φ’
2. Add dummy transition for each ‘may’ action / non-mandatory feature
3. Create a new initial process with special action behaviour leading to FTS encoding, whereas signature leads to dummy transitions

Behaviour = behaviour.T1
T1 = pay(may).T2 + free(may).T3
T2 = change.T3
...
T9 = close.T1

Signature = signature.
  free(may).nil + pay(may).nil + ... + open(may).nil + 
  FreeDrinks(may).nil + ... + Tea(may).nil

VMCmodel = Behaviour + Signature
Step 3: definition of valid products in MTS and variability constraints

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Model Transformation (3/4)

Step 3: definition of valid products in MTS and variability constraints

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\text{VMCmodel} = \text{Behaviour} + \text{Signature}
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Model Transformation (4/4)

Step 4: definition of live action sets and introduction of must transitions

We perform two optimisations for model-checking purposes

1. the explicit definition of additional live action sets
2. the transformation of may transitions into must transitions

These help VMC to understand a model’s live states and thereby take full advantage of the specificities of variability-aware ACTL

Constraints {
  free OR pay
  cancel OR soda OR tea
  takeFree OR open
}

Diagram of state transitions and constraints
Model Transformation (4/4)

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Diagram: [Diagram of the model showing transitions and actions]
Soundness of model transformation

Given FTS $S$ and MTS $S'$. Let $\mathbb{[S]}$ denote the set of valid product configurations for $S$. Let $\text{FTS}(S)$ and $\text{MTS}(S')$ denote the set of LTS products of $S$ and $S'$.

**Theorem**

Let $S$ be FTS and $S'$ be MTS obtained by the model transformation.

1. $\exists$ bijection between $\mathbb{[S]}$ and $\text{MTS}(S')$ such that each $p$ in $\mathbb{[S]}$ is associated to an LTS that contains a (dummy) transition with label $F$ for each feature $F$ in $p$ and no transitions labelled with a feature not in $p$.

2. $\text{FTS}(S)$ and the set of LTS obtained by omitting the dummy transitions from the LTS in $\text{MTS}(S')$ are equal.

**Proof.**

Sketch in SEFM’15 paper. Full proof in forthcoming journal paper.
Recall: Action-based CTL

Syntax over *action formulas* (boolean compositions of actions, denoted by $\psi$), *state formulas* ($\phi$) and *path formulas* ($\pi$)

\[
\phi ::= \text{true} \mid \neg \phi \mid \phi \land \phi \mid [\psi] \phi \mid \langle \psi \rangle \phi \mid E \pi \mid A \pi \mid \\
\quad \mu Y.\phi(Y) \mid \nu Y.\phi(Y) \\
\pi ::= X \{\psi\} \phi \mid [\phi \{\psi\} U \{\psi'\} \phi'] \mid [\phi \{\psi\} W \{\psi'\} \phi'] \mid \\
\quad [\phi \{\psi\} U \phi'] \mid [\phi \{\psi\} W \phi'] \mid F \phi \mid F \{\psi\} \phi \mid G \phi
\]

($Y$ is a propositional variable, $\phi(Y)$ is syntactically monotone in $Y$)

$\mu$ and $\nu$: recursion ("finite looping"/"liveness" and "looping"/"safety")

$X, U, W, F$: action-based neXt, Until, Weak until, Future ("eventually")
Recall: Action-based CTL

Syntax over action formulas (boolean compositions of actions, denoted by $\psi$), state formulas ($\phi$) and path formulas ($\pi$)

$$\phi ::= \text{true} \mid \neg \phi \mid \phi \land \phi \mid [\psi] \phi \mid \langle \psi \rangle \phi \mid E \pi \mid A \pi \mid \mu Y. \phi(Y) \mid \nu Y. \phi(Y)$$

$$\pi ::= X \{\psi\} \phi \mid [\phi \{\psi\} U \{\psi'\} \phi'] \mid [\phi \{\psi\} W \{\psi'\} \phi'] \mid [\phi \{\psi\} U \phi'] \mid [\phi \{\psi\} W \phi'] \mid F \phi \mid F \{\psi\} \phi \mid G \phi$$

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\]

\[
\pi ::= X \{\psi\} \phi \mid \{\phi \{\psi\} U \{\psi'\} \phi'\} \mid \{\phi \{\psi\} W \{\psi'\} \phi'\} \mid \\
\{\phi \{\psi\} U \phi'\} \mid \{\phi \{\psi\} W \phi'\} \mid F \phi \mid F \{\psi\} \phi \mid G \phi
\]

($Y$ is a propositional variable, $\phi(Y)$ is syntactically monotone in $Y$)

$\mu$ and $\nu$: recursion ("finite looping"/"liveness" and "looping"/"safety")

$X$, $U$, $W$, $F$: action-based neXt, Until, Weak until, Future ("eventually")
v-ACTL: variability-aware, action-based CTL

Action formulae $\psi$, state formulae $\phi$, path formulae $\pi$

$$
\psi ::= true \mid a \mid a(e) \mid \neg \psi \mid \psi \land \psi
$$

$$
\phi ::= true \mid \neg \phi \mid \phi \land \phi \mid \langle \chi \rangle \phi \mid [\chi] \phi \mid E\pi \mid A\pi \mid \mu Y.\phi(Y) \mid \nu Y.\phi(Y)
$$

$$
\pi ::= [\phi \{\chi\} U \{\chi'\} \phi'] \mid [\phi \{\chi\} U \phi] \mid [\phi \{\chi\} W \{\chi'\} \phi'] \mid [\phi \{\chi\} W \phi] \mid X\{\chi\} \phi \mid F\phi \mid F\{\chi\} \phi \mid G\phi
$$

$\langle \chi \rangle \Box \phi$: a next state exists, reachable by a must transition executing an action satisfying $\chi$, in which $\phi$ holds

$[\chi] \Box \phi$: in all next states reachable by a must transition executing an action satisfying $\chi$, $\phi$ holds

$F\Box \{\chi\} \phi$: there exists a future state, reached by an action satisfying $\chi$, in which $\phi$ holds and all transitions until that state are must transitions
Informal semantics of ν-ACTL

\[[\psi] \phi\] in all next states reachable by a \textit{may} transition executing an action satisfying \(\psi\), \(\phi\) holds

\[[\psi]\Box \phi\] in all next states reachable by a \textit{must} transition executing an action satisfying \(\psi\), \(\phi\) holds

\(\langle \psi \rangle \phi \equiv \neg [\psi] \neg \phi\) a next state exists, reachable by a \textit{may} transition executing an action satisfying \(\psi\), in which \(\phi\) holds

\(\langle \psi \rangle \Box \phi \equiv \neg [\psi] \Box \neg \phi\) a next state exists, reachable by a \textit{must} transition executing an action satisfying \(\psi\), in which \(\phi\) holds

(\(\langle \psi \rangle \Box\) and \([\psi]\Box\) represent the classic \textit{deontic} modalities \(O\) and \(P\))
Informal semantics of v-ACTL

\[ [\psi] \phi \text{ in all next states reachable by a } \text{may} \text{ transition executing an action satisfying } \psi, \phi \text{ holds} \]

\[ [\psi] \Box \phi \text{ in all next states reachable by a } \text{must} \text{ transition executing an action satisfying } \psi, \phi \text{ holds} \]

\[ \langle \psi \rangle \phi \equiv \neg [\psi] \neg \phi \text{ a next state exists, reachable by a } \text{may} \text{ transition executing an action satisfying } \psi, \text{ in which } \phi \text{ holds} \]

\[ \langle \psi \rangle \Box \phi \equiv \neg [\psi] \Box \neg \phi \text{ a next state exists, reachable by a } \text{must} \text{ transition executing an action satisfying } \psi, \text{ in which } \phi \text{ holds} \]

(\langle \psi \rangle \Box \text{ and } [\psi] \Box \text{ represent the classic deontic modalities } O \text{ and } P)
Informal semantics of $\nu$-ACTL

$[\psi] \phi$ in all next states reachable by a \textit{may} transition executing an action satisfying $\psi$, $\phi$ holds

$[\psi]^{\square} \phi$ in all next states reachable by a \textit{must} transition executing an action satisfying $\psi$, $\phi$ holds

$\langle \psi \rangle \phi \equiv \neg [\psi] \neg \phi$ a next state exists, reachable by a \textit{may} transition executing an action satisfying $\psi$, in which $\phi$ holds

$\langle \psi \rangle^{\square} \phi \equiv \neg [\psi]^{\square} \neg \phi$ a next state exists, reachable by a \textit{must} transition executing an action satisfying $\psi$, in which $\phi$ holds

($\langle \psi \rangle^{\square}$ and $[\psi]^{\square}$ represent the classic \textit{deontic} modalities $O$ and $P$)
Informal semantics of v-ACTL (cont.)

A full path is a path that cannot be extended further (q ··· or q $\not\rightarrow$)

- $E \pi$ there exists a full path on which $\pi$ holds
- $A \pi$ on all possible full paths, $\pi$ holds

- $F \phi$ there exists a future state in which $\phi$ holds
- $F^\square \phi$ there exists a future state in which $\phi$ holds and all transitions until that state are must transitions
- $F \{\psi\} \phi$ there exists a future state, reached by an action satisfying $\psi$, in which $\phi$ holds
- $F^\square \{\psi\} \phi$ there exists a future state, reached by an action satisfying $\psi$, in which $\phi$ holds and all transitions until that state are must transitions

- $G \phi \equiv \neg F \neg \phi$ the path is a full path on which $\phi$ holds in all states
- $AG \phi \equiv \neg EF \neg \phi$ in all states on all paths, $\phi$ holds
Informal semantics of v-ACTL (cont.)

A full path is a path that cannot be extended further \((q \cdots \text{ or } q \not\rightarrow)\)

- \(E\pi\) there exists a full path on which \(\pi\) holds
- \(A\pi\) on all possible full paths, \(\pi\) holds

- \(F\phi\) there exists a future state in which \(\phi\) holds
- \(F\Box\phi\) there exists a future state in which \(\phi\) holds and all transitions until that state are must transitions
- \(F\{\psi\}\phi\) there exists a future state, reached by an action satisfying \(\psi\), in which \(\phi\) holds
- \(F\Box\{\psi\}\phi\) there exists a future state, reached by an action satisfying \(\psi\), in which \(\phi\) holds and all transitions until that state are must transitions

\(G\phi \equiv \neg F \neg \phi\) the path is a full path on which \(\phi\) holds in all states

\(AG\phi \equiv \neg EF \neg \phi\) in all states on all paths, \(\phi\) holds
Informal semantics of v-ACTL (cont.)

A full path is a path that cannot be extended further (q ··· or q ↗)

\[ E \pi \] there exists a full path on which \( \pi \) holds

\[ A \pi \] on all possible full paths, \( \pi \) holds

\[ F \phi \] there exists a future state in which \( \phi \) holds

\[ F^\square \phi \] there exists a future state in which \( \phi \) holds and all transitions until that state are must transitions

\[ F \{ \psi \} \phi \] there exists a future state, reached by an action satisfying \( \psi \), in which \( \phi \) holds

\[ F^\square \{ \psi \} \phi \] there exists a future state, reached by an action satisfying \( \psi \), in which \( \phi \) holds and all transitions until that state are must transitions

\[ G \phi \equiv \neg F \neg \phi \] the path is a full path on which \( \phi \) holds in all states

\[ AG \phi \equiv \neg EF \neg \phi \] in all states on all paths, \( \phi \) holds
Semantics of ν-ACTL over MTS \((Q, A, \overline{q}, \delta^\Diamond, \delta^\Box)\)

Let \(q \in Q\) and \(\sigma\) a full path (from \(q\)) with \(i\)th state \(\sigma(i)\) and \(i\)th action \(\sigma\{i\}\)

\[
q \models \neg \phi \text{ iff } q \not\models \phi \\
q \models \phi \land \phi' \text{ iff } q \models \phi \text{ and } q \models \phi' \\
q \models [\psi] \phi \text{ iff } \forall q' \in Q \text{ s.t. } q \xrightarrow{a(e)} q' \text{ and } a(e) \models \psi, \text{ we have } q' \models \phi \\
q \models [\psi]^\Box \phi \text{ iff } \forall q' \in Q \text{ s.t. } q \xrightarrow{a(e)} q' \text{ and } a(e) \models \psi, \text{ we have } q' \models \phi \\
q \models E \pi \text{ iff } \exists \text{ full path } \sigma' \text{ from } q : \sigma' \models \pi \\
q \models A \pi \text{ iff } \forall \text{ full path } \sigma' \text{ from } q : \sigma' \models \pi \\
q \models \mu Y.\phi(Y) \text{ iff } \bigvee_{i \geq 0} \phi^i(false) \\
q \models \nu Y.\phi(Y) \text{ iff } \bigwedge_{i \geq 0} \phi^i(true) \\
q \models F \phi \text{ iff } \exists j \geq 1 : \sigma(j) \models \phi \\
q \models F^\Box \phi \text{ iff } \exists j \geq 1 : \sigma(j) \models \phi \text{ and } \forall 1 \leq i < j : (\sigma(i), \sigma\{i\}, \sigma(i + 1)) \in \delta^\Box \\
q \models F \{\psi\} \phi \text{ iff } \exists j \geq 1 : \sigma\{j\} \models \psi \text{ and } \sigma(j + 1) \models \phi \\
q \models F^\Box \{\psi\} \phi \text{ iff } \exists j \geq 1 : \sigma\{j\} \models \psi \text{ and } \sigma(j + 1) \models \phi, \text{ and } \forall 1 \leq i \leq j : (\sigma(i), \sigma\{i\}, \sigma(i + 1)) \in \delta^\Box
Preservation of formulae in $\nu$-ACTL$^\square$/$\nu$-ACTLive$^\square$:

$v$-ACTL$^\square$/$v$-ACTLive$^\square$:

$$\phi ::= \text{false} \mid \text{true} \mid \phi \land \phi \mid \phi \lor \phi \mid [\psi] \phi \mid \langle \psi \rangle \square \phi \mid EF \square \phi \mid EF \{\psi\} \phi \mid AF \square \phi \mid AF \{\psi\} \phi \mid AG \phi \mid AF \phi \mid AF \{\psi\} \phi$$

any formula that is true for MTS, is also true for all products (LTS)

$v$-ACTL$^\neg$:

$$\chi ::= \text{false} \mid \text{true} \mid \chi \land \chi \mid \chi \lor \chi \mid \langle \psi \rangle \chi \mid EF \chi \mid EF \{\psi\} \chi$$

any formula that is false for MTS, is also false for all products (LTS)
Preservation of formulae in v-ACTL\(\square\)/v-ACTLive\(\square\)

\(\text{v-ACTL}\square/\text{v-ACTLive}\square:\)

\[
\phi ::= \text{false} \mid \text{true} \mid \phi \land \phi \mid \phi \lor \phi \mid [\psi] \phi \mid \langle \psi \rangle \square \phi \\
EF \square \phi \mid EF \square \{\psi\} \phi \mid AF \square \phi \mid AF \{\psi\} \phi \mid AG \phi \\
AF \phi \mid AF \{\psi\} \phi
\]

any formula that is true for MTS, is also true for all products (LTS)

\(\text{v-ACTL}^{-}:\)

\[
\chi ::= \text{false} \mid \text{true} \mid \chi \land \chi \mid \chi \lor \chi \mid \langle \psi \rangle \chi \\
EF \chi \mid EF\{\psi\} \chi
\]

any formula that is false for MTS, is also false for all products (LTS)
Live states use SPL-specific information

\[ S \models \phi \Rightarrow S_p \models \phi \ \forall \text{product LTS } S_p \text{ of MTS } S \]

Recall: all (reachable) must transitions are preserved \((\langle \rangle^\square, []^\square, F^\square)\)

Live action sets define live states (not occur as final in any product)

MTS

Assume

\[ p \rightarrow^a q \]
\[ b \rightarrow^r q \]

a OR b

LTS

In any product in which \(p\) occurs,
\(p\) has at least one outgoing transition

\[ \Rightarrow p \text{ is a live state, since } a \text{ OR } b \text{ gives rise to a live action set } \{a, b\} \]
Live states use SPL-specific information

\[ S \models \phi \Rightarrow S_p \models \phi \quad \forall \text{ product LTS } S_p \text{ of MTS } S \]

Recall: all (reachable) must transitions are preserved (\(\langle\rangle^\square, \text{[]}^\square, F^\square\))

Live action sets define live states (not occur as final in any product)

\[ \text{MTS} \quad p \xrightarrow{a} q \quad \text{Assume} \quad a \text{ OR } b \]

In any product in which \(p\) occurs, \(p\) has at least one outgoing transition

\[ \Rightarrow p \text{ is a live state, since } a \text{ OR } b \text{ gives rise to a live action set } \{a, b\} \]
Live states use SPL-specific information

\[ S \models \phi \Rightarrow S_p \models \phi \forall \text{ product LTS } S_p \text{ of MTS } S \]

Recall: all (reachable) must transitions are preserved (\( \llcorner \), \( \lceil \), \( F \))

Live action sets define live states (not occur as final in any product)

MTS

\[ \sim \overset{a}{\longrightarrow} \overset{b}{\rightarrow} \]

LTS

\[ \sim \overset{a}{\longrightarrow} \overset{b}{\rightarrow} \]

Assume

a OR b

In any product in which \( p \) occurs, \( p \) has at least one outgoing transition

\[ \Rightarrow p \text{ is a live state, since } a \text{ OR } b \text{ gives rise to a live action set } \{a, b\} \]
Live states use SPL-specific information

\[ S \models \phi \Rightarrow S_p \models \phi \quad \forall \text{ product LTS } S_p \text{ of MTS } S \]

Recall: all (reachable) must transitions are preserved (\(\langle \rangle^\square, [], F^\square\))

Live action sets define live states (not occur as final in any product)

In any product in which \(p\) occurs, \(p\) has at least one outgoing transition

\(\Rightarrow p\) is a live state, since a OR b gives rise to a live action set \(\{a, b\}\)
Live states use SPL-specific information

\[ S \models \phi \implies S_p \models \phi \ \forall \text{ product LTS } S_p \text{ of MTS } S \]

Recall: all (reachable) must transitions are preserved (⟨⟩ □, [] □, F □)

Live action sets define live states (not occur as final in any product)

MTS

Assume

a OR b

In any product in which \( p \) occurs,
\( p \) has at least one outgoing transition

⇒ \( p \) is a live state, since a OR b gives rise to a live action set \( \{a, b\} \)
VMC builds on optimization of UMC (input: UML state machines)

Biere, Fantechi, Gnesi, Mazzanti @ Sci. Comput. Program., 2011

VMC: bounded, on-the-fly model checking, providing explanations

Biere, Cimatti, Clarke, Zhu @ TACAS'99

VMC accepts as input a specification in (value-passing) modal process algebra, possibly with additional variability constraints

- interactively explore the model (MTS)
- derive and explore (all) the model’s valid variants (LTS)
- visualize the model/variants graphically as MTS/LTS
- verify v-ACTL properties over MTS/LTS
- interactively explain why a property is (not) satisfied

Model checking of v-ACTL formulae over MTS can be achieved in a complexity that is linear w.r.t. the state space size
Vending Machine: family-based verification

VMC notifies whenever preservation of a verification result is applicable

The Formula:

\([\text{behaviour}] \, \neg E \, \{\text{true} \, \{\text{not tea}\} \, U \, \{\text{serveTea}\} \, \text{true}\}\)

is TRUE

The formula holds for ALL the MTS products

(states generated= 11, computations fragments generated= 10, evaluation time= 0.000644000 sec.)

ACT-UCTL-SoCL-VACTL

\([\text{behaviour}] \, \neg E \, \{\text{true} \, \{\text{not tea}\} \, U \, \{\text{serveTea}\} \, \text{true}\}\)

It is not possible that serveTea occurs without being preceded by tea
VMC lists for each product the action labels of all may transitions that have been preserved (as must transitions) in that product.

Whenever pay occurs, eventually takePaid occurs.
 Specification of the family of coffee machines in VMC

```
VMC v6.2

1  T1  =  euro(may).T2 + dollar(may).T2
2      T2  =  sugar.T3 + no_sugar.T4
3      T3  =  tea(may).T5 + coffee(may).T6 + cappuccino(may).T7
4      T4  =  cappuccino(may).T8 + coffee(may).T9 + tea(may).T10
5      T5  =  pour_sugar.T10
6      T6  =  pour_sugar.T9
7      T7  =  pour_sugar.T8
8      T8  =  pour_milk.T9
9      T9  =  pour_espresso(may).T11 + pour_regular(may).T11
10     T10 =  pour_tea.T11
11     T11 =  take_cup.T1
12
13     net  SYS  =  T1
14
15     Constraints {  
16            euro ALT dollar  
17            cappuccino OR coffee OR tea  
18            dollar EXC tea  
19            cappuccino REQ coffee  
20            coffee REQ (pour_espresso ALT pour_regular)  
21     }
22```

Kandisky 1908
Products of the family of coffee machines derived by VMC

Valid Products of the Family

product_01+euro+tea
product_02+coffee+euro+pour_espresso
product_03+coffee+euro+pour_regular
product_04+coffee+euro+pour_espresso+tea
product_05+coffee+euro+pour_regular+tea
product_06+cappuccino+coffee+euro+pour_espresso
product_07+cappuccino+coffee+euro+pour_regular
product_08+cappuccino+coffee+euro+pour_espresso+tea
product_09+cappuccino+coffee+euro+pour_regular+tea
product_10+coffee+dollar+pour_espresso
product_11+coffee+dollar+pour_regular
product_12+cappuccino+coffee+dollar+pour_espresso
product_13+cappuccino+coffee+dollar+pour_regular
A product of the family of coffee machines derived by VMC
A formula verified by VMC over the family of coffee machines

It is always the case that whenever sugar is chosen, eventually sugar is poured
A formula verified by VMC over the family of coffee machines

It is always the case that whenever sugar can be chosen, also no sugar can be chosen.
A formula verified by VMC over all products of the family

Upon the insertion of a dollar, it might be the case that eventually a cappuccino can be chosen.
Welcome to VMC

Documentation:

Sample code:

Download:

Requirements:
Any browser with frames, javascript, DHTML, SVG support and popup enabled.
E.g. Firefox, Chrome, Safari, Opera are OK
compatibility with Internet Explorer not tested.

Authors:
Franco Mazzanti (http://fmt.isti.cnr.it/~mazzanti), Aldi Sulova

Credits:
Graphics generated with GraphViz (http://www.graphviz.org/)
Graph minimization with MCRL2_it2cconvert
Family of coffee machines specified in VMC

Permitted variability constraints ALTernative, EXCludes, REQuires, and IFF (shorthand for bilateral REQ) hide logic formalization from user
Family/MTS of coffee machines visualized by VMC

The above graph shows the MTS family model evolutions. Dotted edges denote "may" transitions, full edges denote "must" transitions.

View the graph in DOT format or as a PDF pdf picture or as plain SVG data.
MTS model of coffee machine family actually permits a user to buy a cappuccino with a dollar, something which is forbidden for its products by variability constraint $\text{dollar EXC unsugared_cappuccino}$.
The formula expresses that every path through the MTS starting with a dollar insertion, eventually leads to an unsugared cappuccino.
Outcome of a property explained by VMC

The formula:
[dollar] EF <unsugared_cappuccino> true

is FOUND_TRUE in State C1

This happens because
C1 → C2 {euro(optional)}
C1 → C2 {dollar(optional)}
And the evolutions which satisfy the action formula dollar
also satisfy the subformula:
EF <unsugared_cappuccino> true

In particular:
In state C2 the subformula:
EF <unsugared_cappuccino> true is Satisfied.

The formula:
EF <unsugared_cappuccino> true

is FOUND_TRUE in State C2

This happens because
C2 → C4 {no_sugar} /* ... */
and the subformula:
<unsugared_cappuccino> true
is Satisfied in State C4

The formula:
<unsugared_cappuccino> true

is FOUND_TRUE in State C4

This happens because
C4 → C11 {unsugared_cappuccino(optional)}
the transition label satisfies the action expression unsugared_cappuccino
and in State C11 the subformula:
true is Satisfied.

Logic Formula for Family MTS
[dollar] EF <unsugared_cappuccino> true
Products of family of coffee machines derived by VMC

VMC indeed generates all 10 valid products/LTS that are derivable from the family/MTS if the variability constraints are considered.
Outcomes of a property verified over products with VMC

As required, no valid product (i.e., coffee machine) can deliver an (unsugared) cappuccino upon the insertion of a dollar!
Specification of one of the products derived by VMC

Clicking on a product, VMC opens a window with its textual encoding.
View the graph in [DOT](https://www.bluecloudtech.com/dot/) format or as a [PDF](https://www.w3.org/TR/REC-pdf-structure/) pdf picture or as plain [SVG](https://www.w3.org/TR/svg/) data.

The above graph shows the LTS product model evolutions, which by definition contains only full edges.
The formula expresses that in this particular LTS, there exists both a path to a sugared cappuccino and to an unsugared cappuccino.
Outcome property on previous slide explained by VMC

Non-trivial for branching-time temporal logics!
A value-passing modal process algebra

Let \( \mathcal{A} \) be a set of actions, let \( a \in \mathcal{A} \) and let \( L \subseteq \mathcal{A} \).

Processes are built from terms and actions according to the syntax:

\[
N \ ::= \ [P] \\
P \ ::= \ K(e) \mid P / L / P
\]

\([P]\) denotes a closed system, i.e. it cannot evolve on input actions \(a(\text{?}v)\).

\(K(e)\) is a process identifier from set of process definitions of form \(K(v) \equiv T\).

\[
T \ ::= \ nil \mid K(e) \mid A.T \mid T + T \mid [e \bowtie e] T
\]

\(A \ ::= \ a(e) \mid a(\text{may}, e) \mid a(\text{?}v) \mid a(\text{may}, \text{?}v)\)

\(e \ ::= \ v \mid \text{int} \mid e \pm e\)

\(\bowtie \in \{<, \leq, =, \neq, \geq, >\}\), \(v\) is a variable, \(\text{int}\) is an integer, \(\pm \in \{+, -, \times, \div\}\)
Processes

- \textit{nil} terminated process that has finished execution
- \textit{K} process identifier that is used for modelling recursive sequential processes
- \textit{A.P} process that can execute action \textit{A} and then behave as \textit{P}
- \textit{P + Q} process that can non-deterministically choose to behave as \textit{P} or as \textit{Q}
- \textit{P / L/ Q} process formed by the parallel composition of \textit{P} and \textit{Q} (it can synchronize on actions in \textit{L} and interleave others)

We distinguish 	extbf{must} actions \( a \in \delta \square \) and 	extbf{may but not must} actions \( a(may) \in \delta \lozenge \setminus \delta \square \) (each action type is treated differently in the SOS semantics)
Semantics in SOS style over MTS

\[(\text{sys})\]
\[
\frac{P \xrightarrow{a(e)} P'}{[P] \xrightarrow{a(e)} [P']} \]

\[(\text{act□})\]
\[
\frac{\alpha \in \{a(e), a(?v)\}}{\alpha.P \xrightarrow{\alpha} P}
\]

\[(\text{or□})\]
\[
\frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'} \quad \alpha \in \{a(e), a(?v)\}
\]

\[(\text{int□})\]
\[
\frac{P \xrightarrow{\ell} P'}{P / L / Q \xrightarrow{\ell} P' / L / Q} \quad \ell \notin L
\]

\[(\text{par□})\]
\[
\frac{P \xrightarrow{a(e_1)} P' \quad Q \xrightarrow{a(e_2)} Q'}{P / L / Q \xrightarrow{a} P' / L / Q'} \quad a \in L, e_1 = e_2
\]

\[(\text{guard})\]
\[
\frac{e_1 \triangleleft e_2}{[e_1 \triangleleft e_2] P(e_3) \rightarrow P(e_3)}
\]

(similarly in case of may actions and for the remaining operators)
Bike-sharing systems (BSS)

- Simple concept: a user arrives at a docking station, pays for a bike, uses it for a while and returns it to a station
- Multiple benefits: reduction of vehicular traffic (congestion), pollution, energy consumption, etc.

- Docking stations distributed over a city, typically close to other public transportation hubs (e.g. subway and tram stations)
- (Subscribed) users may rent an available bike and drop it off at any station in the city
- To improve the efficiency and the user satisfaction of BSS, the load between the different stations may be balanced
  - incentive schemes (rewards) to change the behavior of users
  - efficient (dynamic) redistribution of bikes between stations
Bike-sharing systems (BSS)

- Simple concept: a user arrives at a docking station, pays for a bike, uses it for a while and returns it to a station
- Multiple benefits: reduction of vehicular traffic (congestion), pollution, energy consumption, etc.
- Docking stations distributed over a city, typically close to other public transportation hubs (e.g. subway and tram stations)
- (Subscribed) users may rent an available bike and drop it off at any station in the city
- To improve the efficiency and the user satisfaction of BSS, the load between the different stations may be balanced
  - incentive schemes (rewards) to change the behavior of users
  - efficient (dynamic) redistribution of bikes between stations
History of BSS

DeMaio @ Journal of Public Transportation, 2009

- 1st generation free BSS introduced in Amsterdam (*witte fietsen*)
- 2nd generation born in Denmark, first large-scale BSS launched in Copenhagen (*Bycyklen*)
- 3rd generation technology-based BSS in > 500 cities worldwide (*Vélib* in Paris: over 20,000 bikes and 1,800 stations; largest in Hangzhou: ±50,000 bikes and 2,000 stations, one every 100m)
- 4th generation BSS are already being developed, incl. movable and solar-powered stations, electric bikes and mobile (i)phone real-time availability applications (*Bycyklen*)

We collaborate with *PisaMo S.p.A.*, an in-house public mobility company of Pisa’s administration that introduced the public BSS *CicloPi* in Pisa in 2013 (currently ±250 bikes and 24 stations), and *Bicincittà S.r.l. di Torino* (supplying and monitoring)
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Behavioral requirements of a BSS

We consider a BSS with $N$ stations and a fleet of $M$ bikes; Each station $i$ has a capacity $K_i$; Redistribution is optional

1. Users arrive at station $i$
2. If a user arrives at a station with no available bike, (s)he leaves
3. Otherwise, (s)he takes a bike and chooses station $j$ to return it
4. If there are less than $K_j$ bikes at station $j$ when (s)he arrives, (s)he returns the bike and leaves
5. If the station is full she chooses another station $k$ and goes there
6. Redistribution of bikes may be asked for and may possibly occur
7. The user rides like this again until (s)he can return the bike

Inspired by Fricker, Gast @ EURO Journal on Transportation and Logistics, 2014
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Value-passing BSS specification

Station(I,N,J,M) = request(I).
   ( [N=0] nobike(I).Station(I,N,J,M) +
      [N>0] bike(I).Station(I,N-1,J,M) ) +
   return(I).Station(I,N+1,J,M) +
   redistribute(may,?FROM,?TO,?K).
      ( [TO = I] Station(I,N+K,J,M) +
         [TO /= I] Station(I,N,J,M) ) +
   [N > M] redistribute(may,I,J,N-M).Station(I,M,J,M)

net STATIONS = Station(s1,1,s2,1) /redistribute/ Station(s2,0,s1,0)

Users(I,J) = request(I).
   ( bike(I).return(J).Users(I,J) +
      nobike(I).Users(I,J) )

net USERS = Users(s1,s2) // Users(s2,s1)

net BSS = STATIONS /request,bike,nobike,return/ USERS
MTS with parameters and values

AG EF\(\Box\) \{bike(s1)\} true
AG EF □ \{bike(s1)\} true
v-ACTL formula: true ∀ products

Products generated by VMC (not needed for verification)