A framework for quantitative modeling and analysis of highly (re)configurable systems

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Abstract—This paper presents our approach to the quantitative modeling and analysis of highly (re)configurable systems, such as software product lines. Different combinations of the optional features of such a system give rise to combinatorially many individual system variants. We use a formal modeling language that allows us to model systems with probabilistic behavior, possibly subject to quantitative feature constraints, and able to dynamically install, remove or replace features. More precisely, our models are defined in the probabilistic feature-oriented language QFLAN, a rich domain specific language (DSL) for systems with variability defined in terms of features. QFLAN specifications are automatically encoded in terms of a process algebra whose operational behavior interacts with a store of constraints, and hence allows to separate system configuration from system behavior. The resulting probabilistic configurations and behavior converge seamlessly in a semantics based on discrete-time Markov chains, thus enabling quantitative analysis. Our analysis is based on statistical model checking techniques, which allow us to scale to larger models with respect to precise probabilistic analysis techniques. The analyses we can conduct range from the likelihood of specific behavior to the expected average cost, in terms of feature attributes, of specific system variants. Our approach is supported by a novel Eclipse-based tool which includes state-of-the-art DSL utilities for QFLAN based on the Xtext framework as well as analysis plug-ins to seamlessly run statistical model checking analyses. We provide a number of case studies that have driven and validated the development of our framework.

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1 INTRODUCTION

SOFTWARE Product Line Engineering (SPLE) is a software engineering methodology aimed at developing, in a cost-effective and time-efficient manner, a family of software-intensive (highly configurable) systems by systematic reuse. Individual products (or variants) share an overall reference (variability) model of the family, but they differ with respect to specific features, which are (user-visible) increments in functionality. The explicit introduction and management of feature-based variability in the software development cycle causes complexity in the modeling and analysis of software product lines (SPLs). There is a lot of recent research on lifting successful high-level algebraic modeling languages and formal verification techniques known from single (software) system engineering, such as process calculi and model checking, to SPLE (cf., e.g., [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]). The challenge is to handle the variability inherent to SPLs, and to highly configurable systems in general, by which the number of possible variants may be exponential in the number of features. This paper presents our approach to this challenge.

Modeling with the FLAN family

We have developed a modeling approach based on a family of process-algebraic specification languages (FLAN [11], PFLAN [12] and QFLAN [13], surveyed in the invited contribution [14]). This family is inspired by the concurrent constraint programming paradigm of [15], its adoption in process calculi [16], and its stochastic extension [17].

In [11], we introduced the feature-oriented language FLAN. In FLAN, a rich set of process-algebraic operators allows one to specify both the configuration (in terms of features) and the behavior (in terms of actions) of products, while a constraint store allows one to specify all common constraints on features known from variability models such as feature models, as well as additional feature-based constraints on actions. The execution of a process is constrained by the store (e.g. to avoid introducing inconsistencies), but a process can also query the store (e.g. to resolve configuration options) or update the store (e.g. to add new features, even at runtime). The main distinguishing modeling feature of FLAN is a clean separation between the configuration and runtime aspects of an SPL.

In [12], we subsequently equipped FLAN with the means to specify probabilistic models of SPLs, resulting in PFLAN. PFLAN adds to FLAN the possibility to equip each action (including those that install an additional feature, possibly at runtime) with a rate, which can represent notions like uncertainty, failure rates, randomisation or preferences. This allows one to model and analyze the likelihood of installing features, the probabilistic behavior of users of products of the SPL and the expected average cost of products, next to probabilistic quantifications of ordinary temporal logic properties.

A fact that emerged during our experimentation with PFLAN was the need to consider a number of further aspects in the specification and analysis of behavioral models of SPLs, such as the staged configurations known from dynamic SPLs [18], [19] (e.g. adding and removing features as well as activating and deactivating features) and rich quantitative constraints (e.g. pricing constraints) over feature attributes reminiscent of [20].

For this purpose, we proposed the feature-oriented language QFLAN [13] as an evolution of probabilistic PFLAN [12]. QFLAN enriches PFLAN with the possibility to not only install but also uninstall and replace features at runtime as well as with advanced quantitative constraint modeling options regarding the 'cost' of features, i.e. feature attributes related to non-functional aspects such as price, weight, reliability, etc. In particular, the novel modeling

options we introduced were (i) quantitative constraints in the form of arithmetic relations among feature attributes (e.g. the total cost of a set of features must be less than a certain threshold); (ii) propositions relating the absence or presence of a feature to such a quantitative constraint (e.g. if a certain feature is present, then the total cost of a set of features must be less than a certain threshold); and (iii) rich action constraints involving such quantitative constraints (e.g. a certain action can be performed only if the total cost of the set of features constituting the product is less than a certain threshold). The uninstallation and replacement of features can be the result of malfunctioning or of the need to install a better version of the feature (e.g. a software update). We will illustrate this in our case studies, as well as the use of each of the above type of quantitative constraints over feature attributes, by providing concrete examples. It is important to note that the above type of quantitative constraints are significantly more complex than the ones that are commonly associated to attributed feature models [20], [21], [22]. We are not aware of any other approach dealing with all of the above aspects in one unifying framework.

Analysis tools for the FLAN family

Our first tool support for the FLAN family was a prototypical implementation of an interpreter in MAUDE [23], which allowed us to conduct analyses on FLAN models, ranging from consistency checking (by means of SAT solving) to model checking. The introduction of PFLAN to model probabilistic aspects led us to develop corresponding tool support. We combined our MAUDE interpreter with the distributed statistical model checker MULTIVESTA [24], which allowed us to estimate the likelihood of specific system configurations and behavior, and thus to measure non-functional aspects such as quality of service, reliability or performance.

When QFLAN was eventually introduced, it became evident that, as feature attributes were typically not Boolean [20], the problem of deciding whether or not a product satisfies an attributed feature model with quantitative constraints, required more general satisfiability-checking techniques than SAT solving. This naturally led us to the use of Satisfiability Modulo Theory (SMT) solvers like Microsoft's Z3 [25], which allowed us to deal with richer notions of constraints like arithmetic ones. In fact, an important contribution of [13] was the integration of SMT solving into our approach, by means of a combination of our MAUDE QFLAN interpreter and Z3. In this paper, we present a complete re-engineering of the tool, in which we reimplemented from scratch our QFLAN simulator using Java. Also, rather than interacting with Z3, we implemented an ad-hoc constraint solver for QFLAN using Java, obtaining a performance improvement quantifiable in more than three orders of magnitude. As we will see, this re-engineering resulted in a mature tool with a modern integrated development environment for QFLAN.

Formally, our statistical model checking approach consists of performing a sufficient (and minimal) number of probabilistic simulations of a system model to obtain statistical evidence (with a predefined level of statistical confidence) of the quantitative properties being verified. Such properties are formulated in MULTIVESTA's property specification language MultiQuaTEx [24]. Statistical model checking offers unique advantages over exhaustive (probabilistic) model checking. First, statistical model checking does not need to generate entire state spaces and hence scales better without suffering from the combinatorial state-space explosion problem typical of model checking. In particular in the context of highly configurable systems, given their possibly combinatorially many variants, this outweighs the main disadvantage of having to give up on obtaining exact results (100% confidence) with exact analysis techniques like (probabilistic) model checking. Second, statistical model checking scales better with hardware resources, since the set of simulations to be carried out can be trivially parallelized and distributed. MULTIVESTA, indeed, can be run on multi-core machines, clusters or distributed computers with almost linear speedup. A further unique advantage of MULTIVESTA is that it can use the same set of simulations for checking several properties at the same time, thus offering even further reductions of computing time.

To the best of our knowledge, we were the first to apply statistical model checking in SPLE in [12]. There are other approaches to probabilistic model checking of SPLs [26], [27], [28], [29], [30], of which the latter comes closest to ours. In [30], the PROFEAT tool is presented. It provides a guarded-command language for modeling families of probabilistic systems and an automatic translation of such SPL models to the (featureless) input language of the probabilistic model checker PRISM [31]. It caters for the activation and deactivation of features at runtime and quantitative constraints over feature attributes. We will come back to this and other related work in Section 7.

Contribution

This paper provides a comprehensive presentation of our approach to the quantitative modeling and analysis of dynamic SPLs and other highly (re)configurable systems. With respect to our previous work on the FLAN family of modeling languages and its tool support, QFLAN is presented as a DSL with advanced Eclipse-based tool support. New higher-level languages have been incorporated to describe system behavior and property specifications. In particular, the designer can now decide to use either the process-algebraic language introduced in previous papers or a declarative rule-based language in the style of guarded command languages. The novel tool support eases the modeling and analysis task by providing an Eclipse editor for QFLAN specifications (complete with auto-completion, error and syntax highlighting, etc.) as well as integrated plug-ins for the analysis. Remarkable novelties of the new tool support are: (i) a reimplementation of the QFLAN interpreter; and (ii) an implementation of an ad-hoc constraint solver for QFLAN. These novel contributions allow us to considerably reduce simulation time. Last but not least, we validate our approach using several case studies. One of them was used in previous work and contributed to shape our approach, while two others show the scalability and versatility of our approach.

Structure of the paper

The paper outline is as follows. Section 2 presents a running example from [13] that we use throughout the paper to illustrate the main concepts of our approach. Section 3 presents the high-level DSL we developed for QFLAN, followed by its Eclipse-based tool support in Section 4. Statistical analysis of QFLAN models with MULTIVESTA is introduced in Section 5, applied to the running example, followed by experimental quantitative analyses of two further case studies in Section 6. Section 7 discusses related work. Section 8 summarizes our contributions and possible future work.

2 RUNNING EXAMPLE: BIKES PRODUCT LINE

We introduce here a case study from [13] that we use as a running example to illustrate the main concepts of our approach and to provide intuitive cases of its possibilities and limitations. The case study stems from a collaboration with Bicincittà S.r.l. (www.bicincitta.com) and PisaMo S.p.A. (www.pisamo.it) in the context of the European project QUANTICOL (www.quanticol.eu). PisaMo is an in-house public mobility company of the Municipality of Pisa responsible for the introduction of the public bike-sharing system *CicloPi* in the city of Pisa two years ago. This bike-sharing system is supplied and monitored by Bicincittà.

To create an attributed feature model of a product line of bikes, we performed requirements elicitation on a set of documents generously shared with us by Bicincittà. This allowed us to extract the main features of the bikes they sell as part of the bike-sharing system, including indicative prices, and to identify their commonalities and variabilities. We then added some features that we found by reading through a number of documents on the technical characteristics and prices of bikes and their components as currently being sold by major bike vendors. The resulting model, which will be presented in the next section, thus has more variability than typical in bike-sharing systems. Indeed, vendors of such systems traditionally allow little variation to their customers (e.g. most vendors only sell bikes with a so-called step-thru frame, a.k.a. open frame or low-step frame, typical of utility bikes instead of considering other kind of frames as we do), in part due to the difficulties of analyzing systems with high variability to provide guarantees on the deployed products and services. We believe that the progress of SPL analysis techniques (including the contribution of this paper) will help the adoption and hence the provision of richer (bike-sharing) systems with more variability. This is confirmed by the feedback we received during recent meetings with representatives of Bicincittà and PisaMo.

3 MODELING WITH QFLAN

The feature-oriented language QFLAN is the most recent member of the FLAN family (FLAN [11], PFLAN [12], QFLAN [13]) of process algebras inspired by the concurrent constraint programming paradigm of [15], its adoption in process calculi [16], and its stochastic extension [17]. QFLAN separates declarative (pre-)configuration from procedural runtime aspects. It does so by using constraint stores, which allow one to specify all common constraints from feature models (and more) in a *declarative* manner, while a rich set of process-algebraic operators allows one to specify the configuration and behavior of product lines in a *procedural* manner. The semantics unifies *static* (pre-configuration) and *dynamic* (runtime) feature selection/installation.

A detailed and rigorous presentation of the syntax and semantics of QFLAN can be found in [13]. In order to make our framework accessible by practitioners, in this paper we present a higher-level DSL for which we offer a modern Eclipse-based integrated development environment. Given that the specifications of such a DSL are automatically encoded in the corresponding process-algebraic terms, the user is not required to have any knowledge on notions and terminology related to process algebras. In the rest of the paper we freely use QFLAN to refer to both the presented DSL and the underlying process algebra.

The core notions of QFLAN are *features* (cf. Section 3.1), *constraints* (cf. Section 3.2) and *processes* (cf. Section 3.3).

3.1 Features

A product line is defined over a set of *features*, here denoted by $\mathcal{F}$, which represent (user-visible) properties or capabilities of products. As in object-oriented programming, it is sometimes convenient to organize features and relate them to each other, possibly in hierarchies, resulting in so-called *feature diagrams*. For the bike product line of our running example, features such as the engine, the basket, the light, etc., are organized in the feature diagram depicted in Fig. 1.

More details on feature diagrams will follow. For now, it is sufficient to know that they allow us to distinguish between *abstract* features and *concrete* ones. For instance, in our running example the wheel will be considered as an abstract feature, with different kinds of wheels (summer, winter, etc.) as concrete features. The complete set of all features of our running example is specified in QFLAN as shown in Listing 1. The declaration of abstract and concrete features is done in separate blocks `abstract features` and `concrete features`, respectively, and consists of a simple enumeration of the features.

```
begin abstract features                                                          1
  Bike Wheels Energy CompUnit Frame Tablet                                       2
end abstract features                                                            3
                                                                                 4
begin concrete features                                                          5
  AllYear Summer Winter Light Dynamo Battery Engine MapsApp NaviApp GuideApp     6
  Music GPS Basket Diamond StepThru Trashed                                      7
end concrete features                                                            8
```

Listing 1. Features of the running example

A *product* is uniquely characterized by a non-empty subset of $\mathcal{F}$, its *installed* features, while a *product family* is characterized by a set of subsets of $\mathcal{F}$ (i.e. a set of products). It is worth noticing the product explosion typical of SPLs. For instance, in our running example the 21 features yield 1,314 different products. As we shall see later, products are subject to constraints such that not all feature combinations yield valid products. Indeed, constraints can partially reduce the number of products (e.g. some bikes may be too expensive, or too heavy, etc.) but not necessarily so much as to mitigate the inherent exponential explosion.

QFLAN allows one to consider attributed features. Each concrete feature can indeed be equipped with a set of non-functional attributes. In our running example, for instance, we consider the attributes `price`, `weight` and `load`, which respectively represent the specific feature's price in euros,

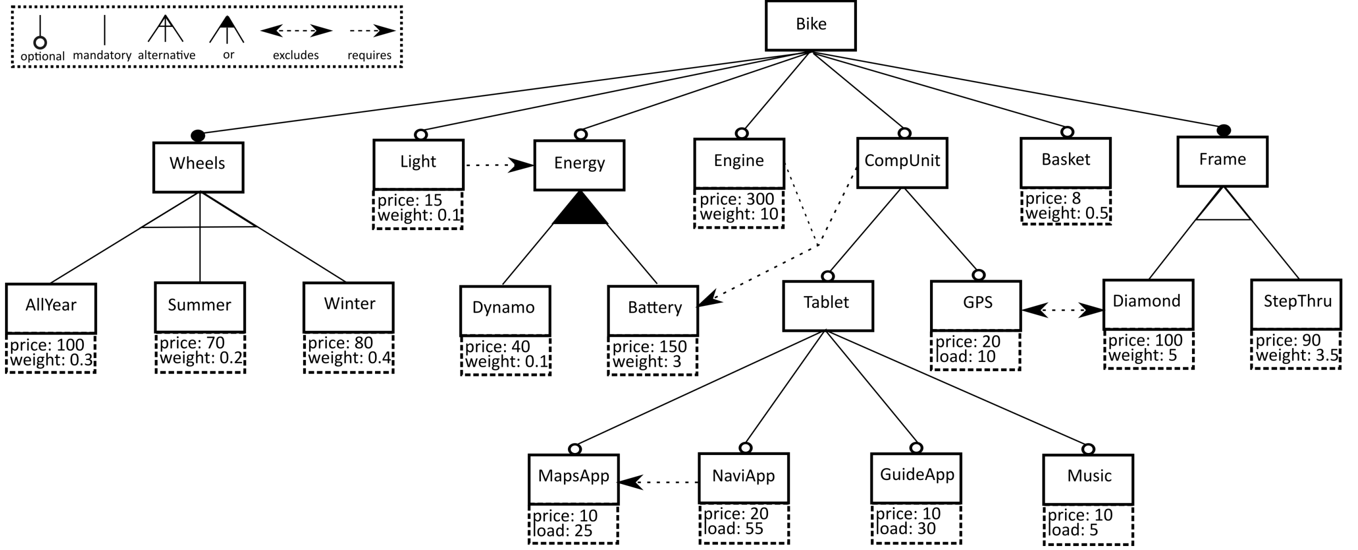


Fig. 1. Attributed feature model of bikes product line

weight in kilos, and computational load in percentage (of system utilization). The complete specification of attributes can be found in Listing 2. The name of the declaration block is `feature predicates` and it includes, for every attribute, the list of concrete features that have the attribute, together with the value of the attribute (if unspecified, by default features have value zero for that predicate). Instead, as we will see later, the attribute values of abstract features are computed automatically from those of the concrete ones.

```
1  begin feature predicates
2    price = { AllYear = 100 , Summer = 70 , Winter = 80 , Light = 15 , Dynamo = 40 ,
3              Battery = 150 , Engine = 300 , MapsApp = 10 , NaviApp = 20 ,
4              GuideApp = 10 , Music = 10 , GPS = 20 , Basket = 8 , Diamond = 100 ,
5              StepThru = 90 }
6    weight = { AllYear = 0.3 , Summer = 0.2 , Winter = 0.4 , Light = 0.1 ,
7               Dynamo = 0.1 , Battery = 3 , Engine = 10 , Basket = 0.5 ,
8               Diamond = 5 , StepThru = 3.5 }
9    load = { MapsApp = 25 , NaviApp = 55 , GuideApp = 30 , Music = 5 , GPS = 10 }
10 end feature predicates
```

Listing 2. Attributes of the running example

3.2 Constraints

The declarative part of QFLAN is represented by a store of constraints on features extracted from the product line requirements plus some additional information (e.g. the mentioned thresholds on maximal cost, weight and load of bikes). QFLAN allows one to specify the following classes of constraints: *hierarchical constraints*, *cross-tree constraints*, *quantitative constraints*, and *action constraints*.

Hierarchical constraints

The standard approach to express feature constraints in the SPL community is by means of a *variability model* which structures the features in the aforementioned *feature diagrams*, possibly enriched with cross-tree constraints and attributes. Such diagrams provide a convenient, visual notation for specifying valid feature combinations. As discussed, the variability model of our bikes example is given in Fig. 1.

In QFLAN such a variability model is specified by providing the feature diagram and its additional cross-tree constraints in separate blocks. For example, the tree-like structure of our running example can be found in Listing 3. The `feature diagram` block has an enumeration of hierarchical relations of the form f `rel` $\{f_1, \ldots, f_n\}$, with `rel` in $\{$`->`, `-OR->`, `-XOR->`$\}$. In particular, f is the father feature node in the tree and $f_1, \ldots, f_n$ are the direct descendent child feature nodes. Features appearing as leaves must be concrete features, while features corresponding to internal nodes must be abstract features.

```
begin feature diagram                                                      1
  Bike -> { Wheels , ?Light , ?Energy , ?Engine , ?CompUnit , ?Basket , Frame }   2
  Wheels -XOR-> { AllYear , Summer , Winter }                              3
  Energy -OR-> { Dynamo , Battery }                                        4
  CompUnit -> { ?Tablet , ?GPS }                                           5
  Frame -XOR-> { Diamond , StepThru }                                      6
  Tablet -> { ?MapsApp , ?NaviApp , ?GuideApp , ?Music }                   7
end feature diagram                                                        8
```

Listing 3. Hierarchical constraints of the running example

The three kinds of relations `rel` correspond to the three kinds of edges appearing in Fig. 1. The *and* relation `->`, used, e.g., to relate `Bike` with `Wheels`–`Frame`, enforces that all descendent features must be present in any product. This constraint can be relaxed by prefixing descendent features with ? (denoted with a circle in Fig. 1, as is common in feature diagrams), to indicate that the specific feature is optional. Descendent features not prefixed by ? are said to be mandatory (black dots). Hence, the relation from `Bike` to `Wheels`–`Frame` imposes any bike to have `Wheels` and `Frame`, while the presence of `Light`, `Energy`, `Engine`, `CompUnit` and `Basket` is optional. The *or* relation `-OR->`, used, e.g., to relate `Energy` with `Dynamo` and `Battery`, enforces that at least one descendent feature must be present in any product. The *xor* relation `-XOR->`, used, e.g., to relate `Wheels` with `AllYear`, `Summer` and `Winter` enforces that precisely one descendent feature must be present in any product.

We remark that the ? prefix can be used only for `->` relations. Moreover, all descendents of optional features must either be reached via `-OR->` or `-XOR->` relations, or be marked as optional.

Also, we assume that only concrete features can be explicitly installed, while abstract features are implicitly installed as soon as one of its descendent feature is installed.

We are now in a position to explain how predicates are evaluated for abstract features: they are computed as the sum of the predicate value of the installed concrete features which descend from the abstract feature. This is very useful, as we can for instance easily refer to the price of an entire bike with price(Bike), or to the computational load of a tablet with load(Tablet).

Cross-tree constraints

The cross-tree constraints of our running example can be found in Listing 4, enumerated under the block cross-tree constraints. It contains two common cross-tree relations. The relation of the form f requires g indicates that whenever feature f (a node in the tree) is installed in a product, then also feature (node) g must be installed, whereas f excludes g indicates that features f and g cannot be both present in the same product.

```
1 begin cross-tree constraints
2   Light requires Energy
3   Engine requires Battery
4   CompUnit requires Battery
5   NaviApp requires MapsApp
6   GPS excludes Diamond
7 end cross-tree constraints
```

Listing 4. Cross-tree constraints of the running example

Quantitative constraints

QFLAN also allows to specify quantitative constraints based on arithmetic relations among feature attributes. For our running example, we consider the following constraints:

(C1) a bike may cost at most 600 euros;
(C2) a bike may weigh up to 15 kilograms;
(C3) a bike's computational load may not exceed 100%.

Constraints (C1)–(C3) are part of the constraint store of our QFLAN model of the bikes product line. As such, they prohibit the execution of any action (e.g. the runtime (un)installation or replacement of features) that would violate these constraints since its execution would result in an inconsistent constraint store. We recall from [13] that the semantics of QFLAN ensures that all executions will end up with a consistent configuration if the process (the procedural part, defined below) begins with a consistent constraint store.

```
1 begin quantitative constraints
2   { price(Bike) < 600 }
3   { weight(Bike) < 15 }
4   { load(Bike) < 100 }
5 end quantitative constraints
```

Listing 5. Quantitative constraints of the running example

Quantitative constraints must be enumerated under the quantitative constraints block. The full specification of the quantitative constraints of our running example is depicted in Listing 5.

Action constraints

As discussed, QFLAN specifications consist of a declarative part, and of a procedural (or behavioral) part. Behavior is expressed in terms of *actions* which are executed. We distinguish between *ordinary actions* and *special actions*. Ordinary actions include one action per feature, modeling the fact that a feature is being used (e.g., execution of Music models the fact that the biker turned on the music). The set of ordinary actions can be complemented defining additional actions in the block actions. The set of additional actions defined for our running example, given in Listing 6, contains for instance the actions sell and dump which model, respectively, the selling and dumping of a bike.

```
begin actions                                                          1
  sell dump maintain book stop break start assistance deploy           2
end actions                                                            3
```

Listing 6. Additional actions of the running example

Special actions instead consist of: install(f) (dynamic installation of a feature f), uninstall(f) (dynamic uninstallation of a feature f), replace(f, g) (dynamic replacement of feature f by g) and ask(K) (to query the store for the validity of constraint/query K).

QFLAN admits a class of *action constraints*, reminiscent of featured transition systems (FTS) [3]. In an FTS, transitions are labeled with actions and with Boolean constraints over the set of features. We associate arbitrary constraints to actions rather than to transitions (and we moreover add a rate to the actions, discussed below). Each action a can have associated a constraint do(a) $\rightarrow$ p, where p is a Boolean proposition involving has(f), used to denote the presence of f in a product, and (in)equations involving feature predicates. In our DSL, the default action constraint for each feature f is do(f) $\rightarrow$ has(f) and it need not be specified. It ensures that in order to use a feature, it must first be installed.

Action constraints act as a kind of guards to either permit or forbid the execution of actions. In our running example, action constraints are used to forbid selling bikes that cost less than 250 euros (C4) and to forbid dumping bikes that cost more than 400 euros (C5). These constraints can be found in Listing 7, showing that action constraints must be enumerated under the action constraints block.

```
begin action constraints                                               1
  do(sell) -> { price(Bike) > 250 }                                    2
  do(dump) -> { price(Bike) < 400 }                                    3
end action constraints                                                 4
```

Listing 7. Action constraints of the running example

3.3 Processes

The procedural part of QFLAN is provided by specifying a set of process blocks (within a processes diagram block), each of which describes a transition system (like the one in Fig. 2) whose basic operation is that of executing actions. We will illustrate the presentation of the process specification with our running example. The behavior associated to our bikes product line is based on a bike-sharing scenario that we abstracted from the bike-sharing system *CicloPi* (cf. Section 2) with some additional behavior concerning not yet realized features, such as the use of electric bikes and the possible runtime installation of apps. These are features that have been envisioned for 4th generation bike-sharing systems [32], [33], some of which (e.g. Tablet, Engine and Battery) have become reality in the recently deployed *Bycyklen* bike-sharing system in Copenhagen.

Combining processes and constraints

Processes are combined with the constraint store; they influence each other according to the concurrent constraint

programming paradigm [15]: a process may update its store which, in turn, may condition the execution of the actions. The underlying formal semantics is that of a discrete-time Markov chain (DTMC), i.e. a transition system whose transitions are decorated with the probability of taking the transition. Such probabilities are derived from the real-valued rates associated to the actions. Recall that we advocate the use of statistical model checking because in general the DTMC is too large to generate. In the statistical model checking analysis we require initial process fragments where S uniquely characterizes a product (i.e. for each feature f, if S does not explicitly contain $\mathtt{has}(\mathtt{f})$, then it is assumed that it contains $\neg\,\mathtt{has}(\mathtt{f})$). In our running example, we assume that the bikes are pre-configured, containing precisely one of the alternative subfeatures from each of the mandatory features `Wheels` and `Frame`. For example, an initial product from the bikes product line can contain the feature set $\{\mathtt{AllYear}, \mathtt{Diamond}\}$.

Probabilistic processes

A process consists of a set of *states* and a set of transitions to move between states. Each transition is labeled with an action, describing what the system does, and a real-valued rate, describing the propensity of the system in performing that action. These action rates allow one to specify probabilistic aspects of SPL models such as the behavior of the user of a product or the likelihood of installing a certain feature at a specific moment with respect to that of other features. Concretely, action rates are used to compute the probability with which an action is executed, obtained by dividing the rate of the action by the total rate of the actions outgoing from the considered state, restricting to those which are enabled by the current configuration present in the constraint store. Installation, removal and replacement of features are applicable as long as they do not introduce inconsistencies, and as long as they satisfy their action constraints. Instead, ordinary actions are only subject to their action constraints. The semantics of the $\mathtt{ask}(\mathtt{p})$ operation is as in concurrent constraint programming [15]: the execution of the action is prevented until p is satisfied by the store.

The dynamics of our running example is given in terms of one process only, sketched in Fig. 2. For simplicity, the action rates are not depicted in Fig. 2, but they can be found in Listing 8 which provides the entire specification of the process. In FACTORY (e.g. of Bicincittà), features may be installed or replaced (e.g. different wheels or a different frame). At a certain point, the configured bike may be sold (as part of a bike-sharing system), but only if it costs at least 250 euro (to satisfy constraint (C4) on action *sell*), after which it arrives in the DEPOSIT (e.g. of PisaMo). It may then be ready to be deployed as part of the bike-sharing system run from this deposit, or it may first need to be further fine-tuned by (un)installing or replacing factory-installed features. Once it is deployed, it results PARKED in one of the docking stations of the bike-sharing system (e.g. *CicloPi*).

A user may book a PARKED bike, resulting in a MOVING bike. While biking, a user may decide to listen to music or switch on the light, in case the corresponding features have been installed. If a user wants to consult one of the apps (a map, a navigator or a guide), then (s)he first needs to stop biking, resulting in a HALTED bike, from where (s)he may consult an app, before eventually start to bike again or park the bike in a docking station. Unfortunately, the bike may also break, resulting in a BROKEN bike. Hence, assistance from the bike-sharing system exploiter arrives. If the bike can be fixed, it is brought to the DEPOSIT. If the damage is too severe, and the bike has a price of at most 400 euros (to satisfy constraint (C5) on action *dump*), then we dump the bike in the TRASH. At regular intervals, assistance from the bike-sharing system exploiter takes a PARKED bike to the DEPOSIT for maintenance.

```
begin processes diagram
  begin process bikesProcess
    states = factory , deposit , parked , moving , halted , broken , trash
    transitions =
    // Sell the bike from the factory
    factory -( sell , 8 )-> deposit ,
    // Install optional features of the bike in the factory
    factory -( install(GPS) , 6)-> factory ,
    factory -( install(MapsApp) , 10 )-> factory ,
    factory -( install(NaviApp) , 6 )-> factory ,
    factory -( install(GuideApp) , 3 )-> factory ,
    factory -( install(Music) , 20 )-> factory ,
    factory -( install(Engine) , 4 )-> factory ,
    factory -( install(Battery) , 4 )-> factory ,
    factory -( install(Dynamo) , 10 )-> factory ,
    factory -( install(Light) , 10 )-> factory ,
    factory -( install(Basket) , 8 )-> factory ,
    // Replace child features of mandatory features of the bike in the factory
    factory -( replace(AllYear , Summer) , 5 )-> factory ,
    factory -( replace(AllYear , Winter) , 5 )-> factory ,
    factory -( replace(Summer , AllYear) , 10 )-> factory ,
    factory -( replace(Summer , Winter) , 5 )-> factory ,
    factory -( replace(Winter , Summer) , 5 )-> factory ,
    factory -( replace(Winter , AllYear) , 10 )-> factory ,
    factory -( replace(Diamond , StepThru) , 3 )-> factory ,
    factory -( replace(StepThru , Diamond) , 3 )-> factory ,
    // Deploy the bike from the deposit
    deposit -( deploy , 10 , {deploys = (deploys + 1)} )-> parked ,
    // Install optional features of the bike in the deposit
    deposit -( install(GPS) , 6 )-> deposit ,
    deposit -( install(MapsApp) , 10 )-> deposit ,
    deposit -( install(NaviApp) , 6 )-> deposit ,
    deposit -( install(GuideApp) , 3 )-> deposit ,
    deposit -( install(Music) , 20 )-> deposit ,
    deposit -( install(Engine) , 4 )-> deposit ,
    deposit -( install(Battery) , 4 )-> deposit ,
    deposit -( install(Dynamo) , 10 )-> deposit ,
    deposit -( install(Light) , 10 )-> deposit ,
    deposit -( install(Basket) , 8 )-> deposit ,
    // Uninstall optional features of the bike in the deposit
    deposit -( uninstall(GPS) , 6 )-> deposit ,
    deposit -( uninstall(MapsApp) , 10 )-> deposit ,
    deposit -( uninstall(NaviApp) , 6 )-> deposit ,
    deposit -( uninstall(GuideApp) , 3 )-> deposit ,
    deposit -( uninstall(Music) , 20 )-> deposit ,
    deposit -( uninstall(Engine) , 1 )-> deposit ,
    deposit -( uninstall(Battery) , 2 )-> deposit ,
    deposit -( uninstall(Dynamo) , 3 )-> deposit ,
    deposit -( uninstall(Light) , 10 )-> deposit ,
    deposit -( uninstall(Basket) , 8 )-> deposit ,
    // Replace child features of mandatory features (Frame cannot be changed)
    deposit -( replace(AllYear , Summer) , 5 )-> deposit ,
    deposit -( replace(AllYear , Winter) , 5 )-> deposit ,
    deposit -( replace(Summer , AllYear) , 10 )-> deposit ,
    deposit -( replace(Summer , Winter) , 5 )-> deposit ,
    deposit -( replace(Winter , Summer) , 5 )-> deposit ,
    deposit -( replace(Winter , AllYear) , 10 )-> deposit ,
    // Replace Battery with Dynamo, in case the battery is not used
    deposit -( replace(Battery , Dynamo) , 1 )-> deposit ,
    // Behavior of deployed bike
    parked -( book , 10 )-> moving ,
    parked -( maintain , 1 )-> deposit ,
    moving -( stop , 5 )-> halted ,
    moving -( break , 1 )-> broken ,
    moving -( Music , 20 )-> moving ,
    moving -( Light , 20 )-> moving ,
    halted -( start , 5 )-> moving ,
    halted -( break , 1 )-> broken ,
    halted -( Music , 20 )-> halted ,
    halted -( GPS , 10 )-> halted ,
    halted -( GuideApp , 10 )-> halted ,
    halted -( MapsApp , 10 )-> halted ,
    halted -( NaviApp , 10 )-> halted ,
    halted -( Light , 10 )-> halted ,
    broken -( assistance , 10 )-> deposit ,
    broken -( dump , 1 , {trashed=1} )-> trash
  end process
end processes diagram
```

Listing 8. The process defining the dynamics of the running example

As said before, the detailed process specification of the case study can be found in Listing 8. Note the tight correspondence between Listing 8 and its graphical representation in Fig. 2. In particular, it contains one state per node in Fig. 2, and a set of transitions per edge in Fig. 2. When the system is in state `factory` we face a choice, weighted by the rates, among three main activities:

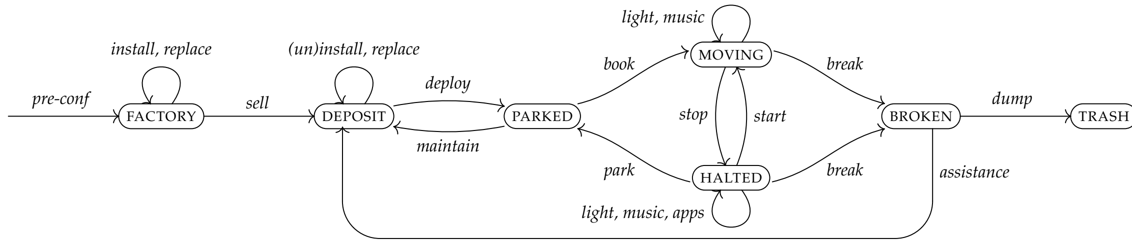


Fig. 2. Sketch of bike-sharing behavior

(1) Sell the bike and send it to the `deposit` (with rate 8). This action can only be executed if (C4) is respected;
(2) Install optional features and iterate on `factory`. The installations are performed only if the constraints are not violated;
(3) Replace pre-installed child features of the mandatory (abstract) features `Wheels` or `Frame`. Again, respecting the constraints.

Note that in (2) we assume that `Music` is the feature installed with higher probability, followed by `MapsApp`, `Dynamo` and `Light`. Note that the semantics of QFLAN forbids the re-installation of installed features (i.e. a product is a set of features, and not a multiset). In (3), we favor the replacement of `Winter` or `Summer` wheels by `AllYear` ones. A frame may be changed as well, but with lower probability.

State `deposit` is similar to `factory`. Clearly, `deposit` differs by the possibility to perform an action `deploy` leading to process `parked`. In addition, `deposit` may also uninstall features, so as to allow for customization. Optional features can be installed and uninstalled with the same rate by `deposit`, except for `Engine`, `Battery` and `Dynamo`, which are uninstalled with a lower rate to penalize their occurrence. This modeling choice is justified by the fact that it is reasonable to assume that uninstalling such features may cost more than installing them. In addition, we assume that the frame identifies the bike that was sold, and thus it cannot be modified in `deposit`. The final action that `deposit` can perform is an interesting one: feature `Battery` can be replaced with the much cheaper `Dynamo`. According to the semantics of QFLAN, this action is performed only if no subfeatures of `CompUnit` or of the `Engine` are currently installed (cf. Fig. 1). This is useful to reduce costs and weight, in case some previously installed feature requiring the battery has by now been uninstalled.

The remaining states `parked`, `moving`, `halted`, `broken` and `trash` are rather simple and are faithful to their informal description above. It is worth to discuss the transitions in Line 28 and Line 76 of Listing 8. In both cases, the transition also updates a *variable*, `deploys` and `trashed`, respectively, used to record that the bike has been deployed or trashed. Indeed, in order to facilitate the modeling within QFLAN, in this paper we enriched it with a set of real-valued *variables*, which can be used in the analysis phase (e.g. to study properties of bikes at first deploy), or in constraints (not shown in our running example, but, e.g., used in the model in Section 6.2). Such variables can be used, e.g., to encode state information or context information (e.g. to model context-aware SPLs [22]). As shown in Listing 9, variables are specified in the `variables` block.

```
1 begin variables
2   deploys = 0
3   trashed = 0
4 end variables
```

Listing 9. Variables of the running example

Note that `factory` is a pure (pre-)configuration state, while `deposit` is not. In fact, parked bikes can be brought back into the deposit, and thus features can be (un)installed or replaced at runtime. This is an example of a staged configuration process, in which some optional features are bound at runtime rather than at (pre-)configuration time.

The initial system configuration is specified in the `init` block. This is provided in Listing 10 for our running example, showing that the initially installed features are `Diamond` and `AllYear`, while the dynamics are given by the process `bikesProcess` (starting in state `factory`, the first state defined in the corresponding `process` block in Line 3 of Listing 8). In case the dynamics of the model under analysis are given in terms of more than one process, this can be specified in `initialProcesses` of the `init` block by listing all required processes (separated by the character `|`). The state of each process will be maintained, and one transition among all those outgoing from all such states will be probabilistically chosen at each step.

```
1 begin init
2   installedFeatures = { Diamond , AllYear }
3   initialProcesses = bikesProcess
4 end init
```

Listing 10. Initial system configuration of the running example

In particular, the modeller is required to start from an initial configuration that satisfies all constraints. This is enforced by an automatic static analysis offered by our tool framework that checks for the validity of all constraints in the initial configuration, and lists those which failed.

The full QFLAN specification of the case study can be found at http://sysma.imtlucca.it/tools/qflan/

4 THE QFLAN TOOL

This section presents the QFLAN tool, a multi-platform application based on Eclipse which enables the modeling and analysis of QFLAN specifications. It does not require any installation process, and it is available together with usage instructions at http://sysma.imtlucca.it/tools/qflan/.

Figure 3 depicts the architecture of the QFLAN tool framework. It is organized in the GUI layer, devoted to modeling aspects, and the core layer, offering support for the analysis of QFLAN specifications.

The components of the GUI layer are shown in Fig. 4. The most notable one is a text editor with editing support

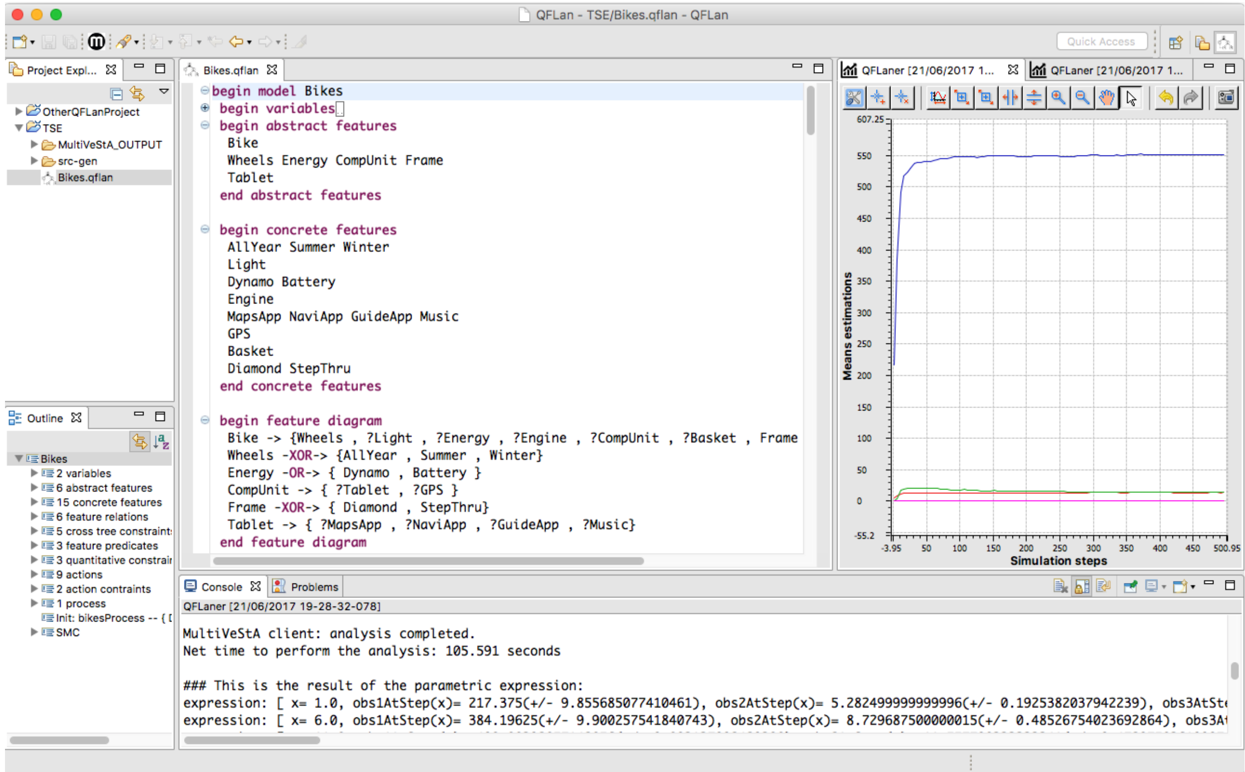


Fig. 4. A screenshot of the QFLAN tool

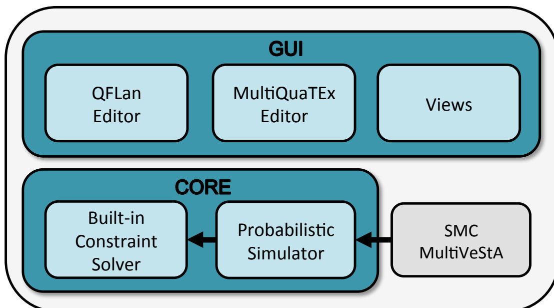


Fig. 3. The architecture of the QFLAN tool

typical of modern integrated development environments (auto-completion, syntax and error highlighting, etc.) developed within the XTEXT framework (top-middle of Fig. 4). For instance, the editor is able to promptly highlight incorrect feature diagrams (e.g. including features with more than one parent or abstract features without descendants) or incorrect feature predicates (e.g. multiple values assigned to one feature). The editor also offers support for the MultiQuaTEx query language, used to analyse QFLAN specifications (cf. Section 5). In addition, the GUI layer offers a number of views, including: a *console view* to display diagnostic information (bottom of Fig. 4); a *project explorer* to handle different QFLAN specifications (left of Fig. 4); and a *plot view* to display analysis results (top-right of Fig. 4).

The main component of the core layer is the probabilistic simulator of QFLAN models. Intuitively, we obtain probabilistic simulations of a QFLAN model by executing it step-by-step starting from the initial configuration specified by the modeler. At each iteration we compute the set of admissible transitions, and select one of them according to the probability distribution resulting from normalizing the rates of the generated transitions. In particular, checking if an action is admissible amounts to checking whether it violates any constraint. This can be established in principle using any SMT solver. However, in order to improve the tool's performance (due to the high number of relatively simple checks performed), we implemented in Java an ad-hoc solver for QFLAN constraints. The reason to do so stems from the fact that the original prototype performed many external invocations of Z3, for relatively simple SMT problems, incurring in a high overhead due to the need to export the current state in Z3 format.

The simulator implements a number of optimizations in order to improve performance by reusing computations performed in previous steps whenever possible. For example, we re-check the admissibility of an action only if the constraint store has been modified in a way that could affect its admissibility.

In [13], [14], we presented a prototypical implementation of a simulator for QFLAN specifications based on Maude and the SMT solver Microsoft Z3, integrated with MULTIVESTA. The QFLAN tool originates from the experience and evidences collected there, but it has been redeveloped from scratch using different, Java-based, technologies. The main outcomes are: (i) a modern integrated development environment for QFLAN; and (ii) an analysis speedup of more than three orders of magnitude, as discussed in Section 5. In addition, the QFLAN tool is no longer a command-line

Linux-based prototype, but a user-friendly multi-platform mature tool with a modern GUI.

5 Statistical Analysis of QFLan Models

In order to perform automated quantitative analysis of QFLan specifications, we integrated the distributed statistical model checker MultiVeStA [24] within the QFLan tool framework.

5.1 MultiVeStA

MultiVeStA can easily be integrated with any formalism that allows probabilistic simulations and it has already been used to analyze a wide variety of systems, including contract-oriented middlewares [34], opportunistic network protocols [35], online planning [36], crowd-steering [37], public transportation systems [38], [39], volunteer clouds [40] and swarm robotics [41].

Within the QFLan tool, MultiVeStA can be used to obtain statistical estimations of quantitative properties of QFLan specifications. MultiVeStA provides such estimations by means of distributed analysis techniques known from statistical model checking (SMC) [42], [43].

Classical SMC allows one to perform analyses like "is the probability that a property holds greater than a given threshold?" or "what is the probability that a property is satisfied?". In addition, MultiVeStA also allows one to estimate the expected values of properties that can take on any value from $\mathbb{R}$, like "what is the average cost/weight/load of products configured according to an SPL specification?". Estimations are computed as the mean of n samples obtained from n independent simulations, with n large enough (but minimal) to grant that the size of the $(1 - \alpha) \times 100\%$ *confidence interval* (CI) is bounded by δ. In other words, if MultiVeStA estimates the value of a property as $\overline{x} \in \mathbb{R}$, then with probability $(1 - \alpha)$ its actual expected value belongs to the interval $[\overline{x} - \delta/2, \overline{x} + \delta/2]$. A CI is thus specified in terms of two parameters: α and δ.

5.2 MultiQuaTEx

MultiVeStA allows to estimate properties written in its property specification language MultiQuaTEx, which extends QuaTEx [44] as described in [24]. MultiQuaTEx is very flexible, based on the following ingredients: real-valued observations on the system states (e.g. the total cost of installed features), arithmetic expressions and comparison operators, if-then-else statements, a one-step next operator (which triggers the execution of one step of a simulation) and recursion. Intuitively, we can use MultiQuaTEx to associate a value from $\mathbb{R}$ to each simulation and subsequently use MultiVeStA to estimate the expected value of such number (in case this number is 0 or 1 upon the occurrence of a certain event, we thus estimate the probability of such an event to occur). For this reason, and for the fact that any level of nesting of recursion and the next operator is allowed, it is known that MultiQuaTEx is strictly more expressive than Probabilistic CTL (cf. [44] for an in-depth discussion on the relation of QuaTEx with other logics).

As depicted in Fig. 3, the QFLan tool offers a MultiQuaTEx editor. Actually, similarly to what we have done for the procedural (or behavioral) part of QFLan, a user is not exposed to MultiQuaTEx. Rather, (s)he is required to express the properties of interest using an intuitive high-level language, from which MultiQuaTEx expressions are automatically generated. As discussed later, the editor restricts to three (large) classes of MultiQuaTEx expressions which where enough to perform all analyses described in this paper. In the future, we might further extend the class of MultiQuaTEx expressions that can be generated using the GUI. Alternatively, the user can provide the path of a file containing a MultiQuaTEx expression (using a graphical file selector). In the following, we will show how to express properties within the QFLan tool, while we refer to [24], [37] for an in-depth presentation of MultiQuaTEx, and to [45] for details on how to directly express properties in MultiQuaTEx.

5.3 Properties of interest to our bikes case study

As done in [13], the analysis capabilities of our framework are exemplified using three families of properties of interest to our case study:

- (P_1) Average price, weight and load of a bike when it is deployed for the first time, or as time progresses;
- (P_2) For each of the 15 concrete features that appear as leaves in the feature model of Fig. 1, the probability to have it installed when a bike is deployed for the first time, or as time progresses;
- (P_3) The probability for a bike to be dumped.

When analyzed at the first deployment of a bike, P_1 and P_2 are useful for studying a sort of *initial scenario*, in order to estimate the required initial investments and infrastructures. For instance, bikes with a high price and a high load (i.e. with a high technological footprint) or equipped with a battery might require docking stations with specific characteristics, or they might have to be collected for the night to be stored safely. Instead, analyzing P_1 and P_2 as time progresses provides an indication of how those values evolve, e.g. to estimate the average value in euros of a deployed bike and the monetary consequences of its loss.

From a more general perspective, properties like P_2 and P_3 measure how often (on average) a feature is actually installed in a product from a product line or how often (on average) a bike is dumped in the trash. The outcome of the former provides important information for those responsible for the production or programming of a specific feature or system module.

Encoding of P_1 and P_2 at first deployment

Listing 11 depicts the code snippet to be specified within the QFLan tool in order to evaluate P_1 and P_2 at a bike's first deployment. These are examples of state properties to be evaluated in a given state of each performed simulation, i.e. the first state met that satisfies the condition specified by the keyword `when`, shown in Line 2. As discussed in Section 3.3, `deploys` is a variable that is increased when the bike is deployed from the deposit to a parking lot. The `when` clause can be given as a Boolean expression involving the presence/absence of a given feature, and (in)equations on the value of a predicate or of a variable (e.g. `deploys > 0` in

Line 2). Property P_1 is specified in Line 3, where we specify that we want to study the average price, weight and load of bikes. In addition, we also query the expected number of simulation `steps` to perform the first deployment. In square brackets we specify the δ to be used for these properties. Instead, Line 4 corresponds to P_2. In fact, by providing the name of a feature we query the QFLAN tool to study the probability of having it installed in the state of interest. In particular, a state property can be any arithmetic expression involving `step`, a feature, a variable or a predicate.

```
1 begin analysis
2 query = eval when {deploys > 0} :
3   { price(Bike)[delta = 20] , weight(Bike)[delta = 1] , load(Bike)[delta = 5] ,
        steps[delta = 1] ,
4     AllYear , Summer , Winter , GPS , MapsApp , NaviApp , GuideApp , Music ,
          Diamond , StepThru , Battery , Dynamo , Engine , Basket , Light
5   }
6 default delta = 0.1
7 alpha = 0.1
8 parallelism = 4
9 end analysis
```

Listing 11. $\mathbf{P_1}$ and $\mathbf{P_2}$ at first deployment

For the state observations of Line 4, the default value of δ provided in Line 6 is used, while all properties share the same alpha specified in Line 7. Finally, the keyword `parallelism` in Line 8 locally distributes the simulations across four distinct Java processes (which will be allocated on different cores, if possible, by the JVM). All experiments described in this paper use value 4 for the parameter `parallelism`. By design choice, viz. to have a stand-alone easy-to-use application, we do not use the ability of MULTIVESTA to distribute simulations across different machines. However, no technical reason prevents us from extending our tool in this way in the future.

Notably, Listing 11 shows how MultiQuaTEx allows one to express more properties at once (in this case 19) which are estimated by MULTIVESTA reusing the same simulations. Furthermore, more queries can be expressed, each with its `when` clause, and list of state observations, again all evaluated reusing the same simulations. We remark that a procedure taking into account that each property might require a different number of simulations is adopted to satisfy the given confidence interval CI.

In Section 6.1 we will see a variant of properties as in Listing 11 with the keyword `when` replaced with `until`. Intuitively, `until` checks that a Boolean property holds in all simulation states met until a given condition holds.

Encoding of P_1–P_3 as time progresses

We now discuss how to express variants of P_1 and P_2 as well as P_3 measured as time progresses, demonstrating how to analyze properties upon the variation of a parameter, in this case the number of performed simulation steps. Listing 12 shows the code snippet necessary to analyze such properties. Essentially, the only required change (cf. Line 2 of Listing 11 and Line 2 of Listing 12) is to substitute the keyword `when` with: (i) the parameter of interest, specified by the keyword `for`, followed by a variable, a predicate or a feature (in this case the variable step); and (ii) the values of interest for the parameter (starting `from` an initial value, up `to` a final value, `by` a given increment). This is shown in Line 2, specifying values 1, 6, 11, ..., 496. As we will see, this interval is reasonable, since all studied properties tend to stabilize within this interval. As for Listing 11, Lines 3–4 correspond to properties P_1 and P_2, respectively. Instead, Line 5, corresponding to P_3, concerns the probability to dump the bike. In fact, as discussed in Section 3.3, `trashed` is a variable set to 1 when the bike is dumped.

TABLE 1
Property P_1 evaluated at a Bike's first deployment, and average number of steps required for deployment

Constraints		Steps to deploy	Feature attributes (P_1)		
C1	C2		Price	Weight	Load
600	15	12.19	380.92	8.10	20.92
800	20	13.33	508.89	12.46	21.74

```
begin analysis                                                                      1
query = eval for step from 1 to 500 by 5 :                                          2
  { price(Bike)[delta = 20] , weight(Bike)[delta = 1] , load(Bike)[delta = 5] ,     3
    AllYear , Summer , Winter , ... , Basket , Light ,                              4
    trashed                                                                         5
  }                                                                                 6
default delta = 0.1                                                                 7
alpha = 0.1                                                                         8
parallelism = 4                                                                     9
end analysis                                                                        10
```

Listing 12. $\mathbf{P_1}$–$\mathbf{P_3}$ for varying simulation steps

5.4 Statistical analysis of our bikes case study

We now report on the evaluation of the discussed properties. All experiments were performed on a laptop equipped with a 2.4 GHz Intel Core i5 processor and 4 GB of RAM, distributing the simulations among its 4 cores (i.e., setting `parallelism` to 4). We compare the obtained runtimes with those required by the previous prototypical implementation of QFLAN based on Maude and Microsoft's Z3, as used in [13]. We do not explicitly report the results obtained for the latter, as they are in line with those computed with the new implementation.

Evaluation of P_1 and P_2 at first deployment

The analysis of Listing 11 required 480 simulations, performed in about 8 seconds. In particular, `steps` is the property that required more simulations, viz. 480, while `price` required only 160 simulations. Instead, the previous version of the tool required about 20 minutes. The results are shown in the first row of Tables 1 and 2. Notably, the probability of installing an engine is very low, estimated at 0 (i.e. with probability 0.9 it belongs to $[0, 0.05]$, according to the specified CI). We guess that this is due to constraints (C1) and (C2), imposing bikes to cost less than 600 euros, and weighing less than 15 kilos. In fact, the estimated average price and weight of bikes at first deployment is 380.92 euros and 8.1 kilos, respectively, while an engine costs 300 euros and weighs 10 kilos. In order to confirm this hypothesis, we analyzed the same property in a new model in which (C1) and (C2) allow bikes to cost at most 800 euros and weigh at most 20 kilos. The results, shown in the second row of Tables 1 and 2, confirm our hypothesis. This analysis thus revealed that the constraints were in disagreement with the quantitative attributes of the features. The latter analysis required 1,200 simulations, performed in about 10 seconds. In this case the estimation of the average price required 1,200 simulations rather than 160 as in the first case (while the previous version of the tool required about 20 minutes). This is because the looser constraints of the latter analysis

TABLE 2
Property P_2 evaluated at a Bike's first deployment

Constraints		Concrete features (P_2)														
C1	C2	AllYear	Summer	Winter	Light	Dynamo	Battery	Engine	MapsApp	NaviApp	GuideApp	Music	GPS	Basket	Diamond	StepThru
600	15	0.55	0.26	0.22	0.54	0.77	0.91	0.0	0.27	0.04	0.29	0.47	0.04	0.67	0.66	0.34
800	20	0.57	0.24	0.20	0.59	0.74	0.89	0.44	0.24	0.06	0.27	0.50	0.10	0.62	0.73	0.27

induce a higher variability of bike prices. In fact, the installation of an engine, the most expensive among the considered features, results in a steep increase of bike prices.

Evaluation of P_1–P_3 as time progresses

We evaluated the property of Listing 12 for our case study. We report the results obtained for the model in which (C1) and (C2) bound the price and weight of a bike to 800 and 20, respectively. All such analyses (19×25 different properties) were evaluated using the same simulations. Overall, 1,200 simulations were needed, performed in about 80 seconds. The previous implementation required about 70 minutes. The results are presented in four plots in Fig. 5: one for the average price (a), one for the average weights and loads (b), one for the probabilities of installing features (c) and one for the probability of dumping the bike (d).

Fig. 5a shows that the average price (on the y-axis) of the intermediate bikes generated from the product line at step 1 is 230, hence higher than the 200 euros of the initial configuration (with AllYear and Diamond installed). In particular, it is possible to see an initial fast growth of the price until reaching an average price of about 510 euros, after which the growth slows down, reaching about 540 euros at step 100 and 543 at step 500. This is consistent with our QFLAN specification, which has a pre-configuration phase (FACTORY) during which a number of features can be installed, followed by a customization phase (DEPOSIT), where features can be (un)installed and replaced. We recall that FACTORY does not perform any uninstalling, while we note that the uninstalling actions of DEPOSIT do not introduce decrements of the price, on average. A manual inspection of the data revealed that the phase of fast growth that slows down in the observed steps 11 and 16. This is consistent with the analysis described in the second row of Table 1, where the average number of steps to complete the first DEPOSIT phase is estimated as being close to 13. In addition, the average price at time step 13 shown in Table 1 is within the prices observed at steps 11 and 16, respectively 492 and 515. Note, finally, that the probability of a bike to return to the DEPOSIT after its first deployment is quite low. In fact, as specified in Listing 8, PARKED has a transition with rate 10 towards MOVING and one with rate 1 towards DEPOSIT. Thus, in average, the price of bikes is only slightly affected by (un)installations and replacements performed by successive DEPOSIT phases.

Fig. 5b shows that weight evolves similarly to price: a first phase of fast growth is followed by a slower growth. As regards the load, instead, after the phase of fast growth up to 23%, it slowly decreases to stabilize around 14%.

As confirmed by Fig. 5c, the probabilities (on the y-axis) for each of the features to be installed evolve similarly to the average price and weight of the generated products, although, clearly, with different scales. It is interesting to note that the pre-installed features AllYear and Diamond have high probability of being installed at step 1, after which the probability decreases during the first 16 steps. The dashed lines refer to all concrete features descending from CompUnit, which are the only ones with non-zero computational load. The slow decrease of load shown in Fig. 5b is due to the slow decrease in having installed MapsApp, NaviApp and GuideApp, which are the features with highest computational load. Instead, Music, whose probability of being installed remains above 0.5 has only 5% of computational load.

Fig. 5d shows that bikes are dumped with very low probability. The reason is twofold. First, the transition from BROKEN to TRASH has a much lower rate than the one to DEPOSIT, and similarly for those from MOVING and HALTED to BROKEN (cf. Listing 8). Second, the average price of bikes quickly rises above 400 euros (Fig. 5a) and constraint (C5) prohibits dumping bikes costing more than 400 euros.

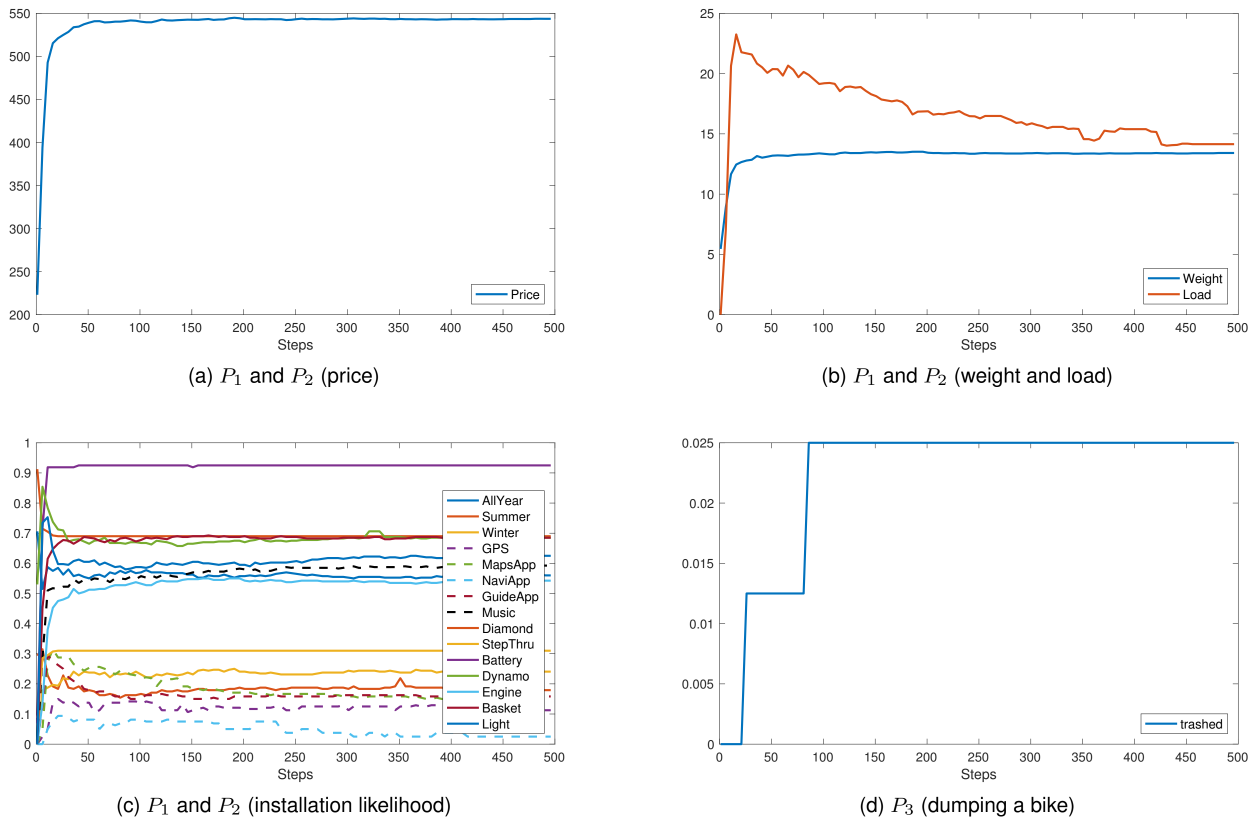
6 EVALUATION

We have used our tool-supported methodology to model and analyze a number of small case studies in our current and previous work, including classical ones from the SPLE literature such as the coffee vending machine [7], [46], [47], [48], [49], [50], [51] as well as novel ones such as the running example of bikes used here. We report in this section on two additional case studies that witness two particular features of our approach, namely scalability of the analysis and flexibility of the modelling language and its analysis support. First, in Section 6.1 we use the classical example of a product line of elevators to evaluate the scalability of our tool support. This case study has been shown to be very demanding in terms of scalability when large sizes of elevator systems are considered (cf., e.g. [30], [52]) and we will demonstrate that we can handle significantly larger instances with respect to existing approaches. Second, in Section 6.2 we model and analyze a novel case study that extends a classical example of risk analysis of a safe lock system, thus illustrating the applicability of our approach also in a non-SPL setting.

6.1 Elevator

The case study we consider here is adapted from the various incarnations of the Elevator product line, originally introduced in [53], which has become a benchmark for SPL analysis (cf., e.g., [9], [20], [30], [52], [54], [55], [56], [57], [58]). This case study is particularly challenging, not so much due to the number of independent, unconstrained features (9, yielding 512 products) but rather due to the need to consider instances with a large number of floors.

The Elevator SPL consists of a number of platform and cabin buttons, one for each platform, which call the elevator.



(a) P_1 and P_2 (price)

(b) P_1 and P_2 (weight and load)

(c) P_1 and P_2 (installation likelihood)

(d) P_3 (dumping a bike)

Fig. 5. Results of measuring P_1–P_3

A button that is pressed (chosen non-deterministically) remains pressed until the elevator has served the floor and opened its doors. Serving a floor means opening and closing its doors. We consider the nine features introduced in [52], [53] that can modify the elevator's behavior:

Anti-prank — Normally, a button will remain pushed until the corresponding floor is served. With this feature, a button has to be held pushed by a person.

Empty — If the lift is empty, then all requests made in the cabin will be cancelled.

Executive floor — One floor has priority over others and is served first, both for cabin and for platform requests.

Open when idle — When idle, the lift opens its doors.

Overload — The lift will not close its doors when overloaded.

Park — When idle, the lift returns to the first floor.

Quick close — The doors cannot be kept open by holding the platform button pushed.

Shuttle — The lift will only change its direction at the first and the last floor.

Two-thirds full — Whenever the lift is two-thirds full, it will serve cabin calls before platform calls.

The core logic of the controller is obviously subject to many constraints (e.g. the doors cannot be open while the elevator is moving) and the activated features add even more constraints (e.g. the Overload feature should impede to close the doors if the cabin is overloaded). Specifying all such constraints in an operational description is rather challenging and results in very sophisticated and cumbersome control flow statements (cf., e.g., [52]).

To show the effectiveness of QFLan, we analyze a classical property of the Elevator SPL from [53] against variations obtained by increasing the number of floors from 5 to 40, while fixing the capacity of the elevator to 8 persons and the maximum allowed load to 4 persons (enforced only if the feature Overload is installed). Instead, the classical approaches mentioned above all restrict to models with less than 10 floors and fewer persons.

In particular, we analyze the property in Listing 13, which establishes that if the number of people in the elevator (the `load` variable) is beyond the capacity of the elevator (the `capacity` variable), then the elevator does not move (`direction == 0.0`). We check this property for all the states met in the first `maxStep` steps, i.e. until the condition `steps < maxStep` holds. In all cases, we obtained a probability equal to 1, because, by construction, the elevator does not move when the current load is higher than the capacity. Hence, all simulations performed in these analyses consisted of exactly `maxStep` steps.

```
begin analysis
  query = eval until {steps < maxStep} : {load >= capacity implies direction == 0.0}
  default delta = 0.1
  alpha = 0.1
  parallelism = 4
end analysis
```

Listing 13. Query to establish a safety property of the elevator

Figure 6 provides the runtimes (in seconds) of analyzing variants of the safety property for different values of `maxStep`, one per trace. In order to reduce stochastic noise, we provide runtimes averaged over 10 independent analyses. The figure provides two kinds of scalability analysis: (1) by focusing on a single trace we can fix the number of

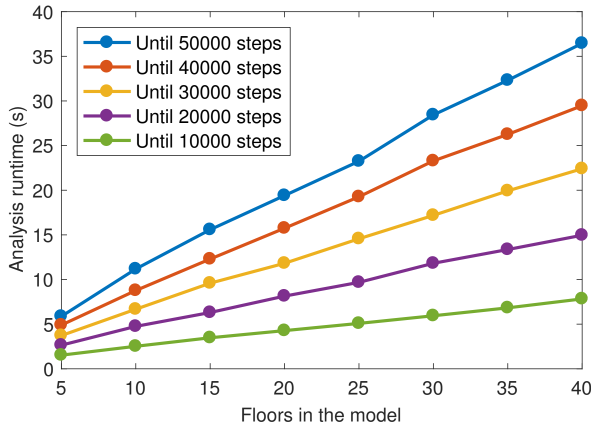


Fig. 6. Runtime in seconds for the analysis of Listing 13; each trace refers to variants of the property for `maxStep` in $\{10, 20, 30, 40, 50\} \times 10^3$

performed simulation steps, and vary the size of the system (the floors); (2) by considering one point of the x-axis (the floors), we can fix the model under analysis, and vary the number of performed simulation steps. In both cases, we note a linear increase in the obtained runtime.

All experiments were performed on a static pre-defined configuration consisting of all features except Park.

6.2 Safe lock

To illustrate the flexibility of our approach to model case studies from different application domains, we show in this section how QFLan can be used to perform risk assessment in security scenarios with high variability. In particular, we focus on the use of attack trees and the seminal example from that area, namely the Safe Lock [59].

Fig. 7(a) presents the original attack tree from [59]. It provides a specification of a risk assessment for a safe lock system. An attack tree is essentially an and/or tree, where nodes represent goals, and sub-trees represent sub-goals. In this case, the root node represents the main threat being analyzed, namely the lock being opened by an attacker. Each of its four children are possible ways of enacting such a threat. The sub-goal Eavesdrop has two sub-goals that need to be accomplished (thus their combination as *and*-children). Nodes are decorated with an estimation of the cost that the attacker would have to pay to succeed in enacting the corresponding action. The classical analysis of such trees is to compute the minimal cost for an attacker to succeed.

Attack trees can easily be modelled as feature diagrams, with the following rationale: a node, which in an attack tree represents a goal, can be modeled as a feature of the system, that the attacker tries to activate. The sub-goal relation is modeled by the feature hierarchy. In particular, the attack tree of our case study can be modeled as in Fig. 7(b). We introduce a slight variation to overcome a well-known limitation of the original attack trees, namely the inability to encode the ordering of events. Indeed, Listen to Conversation should occur before Get Target to State Combo, which we can model with a *requires* cross-tree constraint.

As a matter of fact, the flexibility of the way feature models are specified in QFLAN allows us to specify richer relations among sub-goals. For instance, we can specify that Eavesdrop is only successful if the attacker first listens to a conversation and then gets the target to state the combo, thus refining the original *and*-relation among such sub-goals. This is similar in spirit to the extension of attack trees with sequential conjunction from [60], which imposes orders on the execution of actions in the tree. Further constraints can be imposed on execution, in line with those discussed for the other examples.

A noteworthy advantage of modeling such scenarios with QFLAN is that we can model the behavior of several classes of attackers and study their performance, and thus the robustness of the system against them. To exemplify this, we have considered two attackers, sketched in Fig. 8. Their full process specifications can be found in Listing 14.

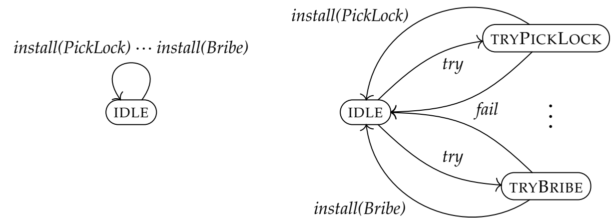


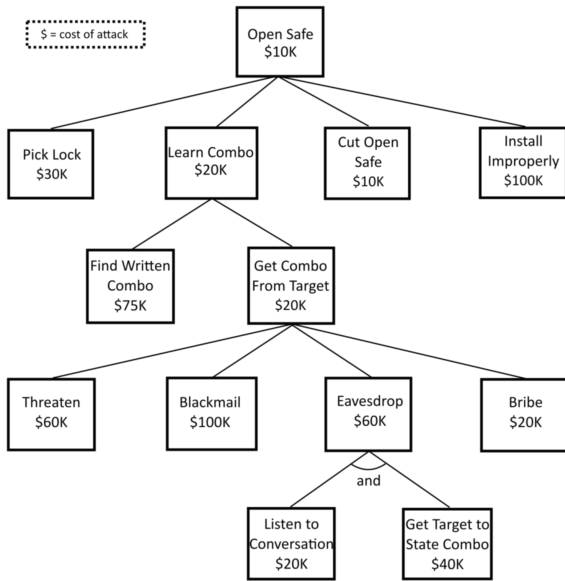
Fig. 8. Behavior of PowerfulAttacker (left) and FailingAttacker (right)

The Powerful attacker always succeeds when trying to achieve a goal and has unlimited resources. Instead, the Failing attacker can fail to successfully achieve a goal and may need several attempts to achieve them. This is modeled using rates. In addition, (s)he has limited resources.

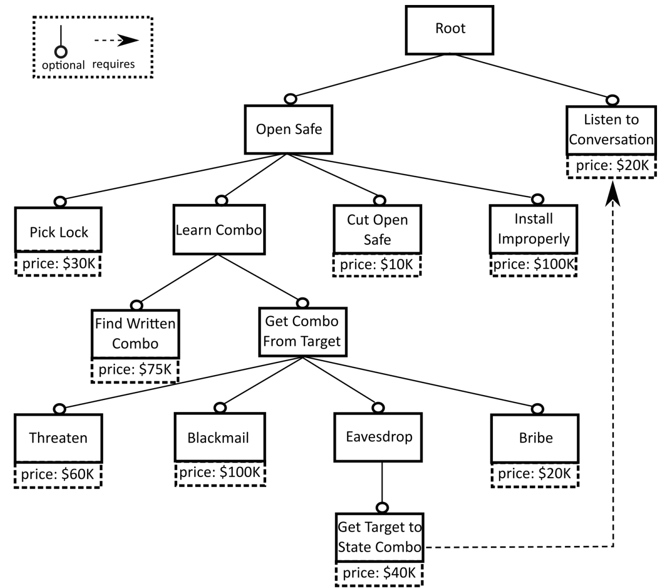
```
begin processes diagram
  begin process powerfulAttacker
    states = idle
    transitions =
    idle -( install(PickLock) , 1 )-> idle ,
    idle -( install(CutOpenSafe) , 1 )-> idle ,
    idle -( install(InstallImproperly) , 1 )-> idle ,
    idle -( install(FindWrittenCombo) , 1 )-> idle ,
    idle -( install(Threaten) , 1 )-> idle ,
    idle -( install(Blackmail) , 1 )-> idle ,
    idle -( install(ListenToConversation) , 1 )-> idle ,
    idle -( install(GetTargetToStateCombo) , 1 )-> idle ,
    idle -( install(Bribe) , 1 )-> idle
  end process

  begin process failingAttacker
    states = idle , tryPickLock , tryCutOpenSafe , tryInstallImproperly ,
          tryFindWrittenCombo , tryThreaten , tryBlackmail ,
          tryListenToConversation , tryGetTargetToStateCombo , tryBribe
    transitions =
    // Try an attack
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryPickLock ,
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryCutOpenSafe ,
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryInstallImproperly ,
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryFindWrittenCombo ,
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryThreaten ,
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryBlackmail ,
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryListenToConversation ,
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryGetTargetToStateCombo ,
    idle -( try , 1 , {cumul_cost = cumul_cost + 1} )-> tryBribe ,
    // Successful attack
    tryPickLock -( install(PickLock) , 1 )-> idle ,
    tryCutOpenSafe -( install(CutOpenSafe) , 1 )-> idle ,
    tryInstallImproperly -( install(InstallImproperly) , 1 )-> idle ,
    tryFindWrittenCombo -( install(FindWrittenCombo) , 1 )-> idle ,
    tryThreaten -( install(Threaten) , 1 )-> idle ,
    tryBlackmail -( install(Blackmail) , 1 )-> idle ,
    tryListenToConversation -( install(ListenToConversation) , 1 )-> idle ,
    tryGetTargetToStateCombo -( install(GetTargetToStateCombo) , 1 )-> idle ,
    tryBribe -( install(Bribe) , 1 )-> idle ,
    // Failed attack
    tryPickLock -( fail , 10 )-> idle ,
    tryCutOpenSafe -( fail , 10 )-> idle ,
    tryInstallImproperly -( fail , 10 )-> idle ,
    tryFindWrittenCombo -( fail , 10 )-> idle ,
    tryThreaten -( fail , 10 )-> idle ,
    tryBlackmail -( fail , 10 )-> idle ,
    tryListenToConversation -( fail , 10 )-> idle ,
    tryGetTargetToStateCombo -( fail , 10 )-> idle ,
    tryBribe -( fail , 10 )-> idle
  end process
end processes diagram
```

Listing 14. Two kind of attackers for the safe lock scenario

Clearly, any reasonable attacker should stop attacking once an attack has been successful. This can be naturally ex-



(a) Schneier's simple attack tree against a physical safe



(b) An attributed feature model representing the attack tree

Fig. 7. Attack tree: (a) redrawn from [59]; (b) feature model representation of it

pressed in QFLAN using the action constraints in Listing 15, which block the attacker after an attack succeeded.

```
begin action constraints
  do(tryAction) -> !has(OpenSafe)
  do(install(...)) -> !has(OpenSafe)
end action constraints
```

Listing 15. Constraints to stop attacks after success

QFLAN's rich specification language allows to express, e.g., further constraints on the accepted classes of attacks. Consider for instance the two constraints in Listing 16. In the first one, we restrict to (successful) attacks that cost less than 100 units (i.e. that install features with less than that price). Instead, the second constraint restricts to attack attempts (independently of their success) that cost less than 10 units. Noteworthy, using the first constraint we restrict the family of admissible products, while the latter constraint regards only the behavioral part of the model.

```
begin quantitative constraints
  // Restrict to attacks that cost less than 100
    { cost(Root) <= 100 }
  // Attacks can fail. Attack attempts have a cost.
  // Restrict to attackers that have a maximum budget.
    { cumul_cost <= 10 }
end quantitative constraints
```

Listing 16. Further constraints to specify the class of accepted attacks

For both attackers, it is interesting to know what is the probability that an attack succeeds in a given amount of time, as well as the average cost of attacks. Such analyses can be performed in QFLAN, as shown in Listing 17. There we query the probability of installing the feature OpenSafe, the cost of the corresponding product and the cost accumulated by the failing attacker while trying to install the features corresponding to the sub-goals. We consider three configurations: (a) a powerful attacker with constraints as specified above; (b) an attacker that might fail with the same constraints; and (c) an attacker that might fail with less resources, obtained by changing the constraint on the `cumul_cost` in Listing 16 to `{ cumul_cost <= 10 }`. This is obtained by running once the analysis on each model variant, each requiring about 12 seconds.

```
begin analysis
  query = eval from 0 to 40 by 1 :
    { OpenSafe [delta = 0.05] , cost(Root) , cumul_cost }
  default delta = 1   alpha = 0.05
end analysis
```

Listing 17. Analysis of the safe lock model

Figure 9 plots the probabilities of successful attacks. We note that the powerful attacker succeeds with probability almost 1 after one step, whereas for the other attacker the probability of success increases slowly. We also note that in case of constraint `{ cumul_cost <= 10 }`, the probability of success stabilizes at about 0.6 after 20 steps. Indeed, according to Listing 14, `cumul_cost` increases by 1 every two steps. Instead, in case `{ cumul_cost <= 20 }`, the probability reaches value 0.8 after 40 steps. However, this is not due to the mentioned constraint. In fact, Fig. 10 plots the costs and the cumulative costs computed for the three model variants. We see that the average cumulative cost (not shown for the powerful attacker, because it is always 0) reaches the threshold 10 after 20 steps (in case `{ cumul_cost <= 10 }`), while it is much lower than 20 in the other case. Hence, the constraint `{ cumul_cost <= 20 }` has less impact on the dynamics. As a last remark, we note that costs evolve similarly to probabilities, even though with different scales.

7 Related Work

We concentrate on related approaches that focus on the application of automated verification techniques, and in particular (probabilistic) model checking, in the specific context of behavioral models of (dynamic) SPLs. We give a brief overview of models for specifying SPL behavior, followed by their associated verification techniques and tools.

7.1 Models

Most better known SPL behavioral modeling languages are based on superimposing multiple LTSs representing vari-

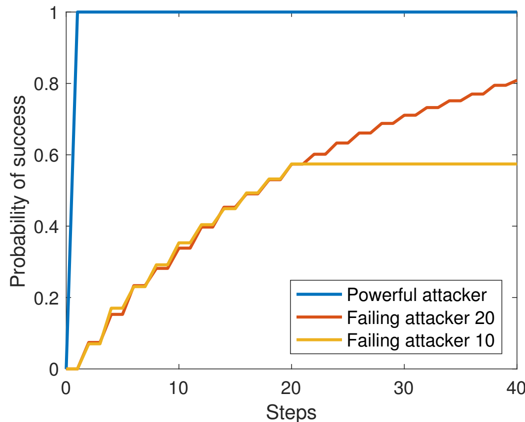


Fig. 9. Probabilities of successful attacks

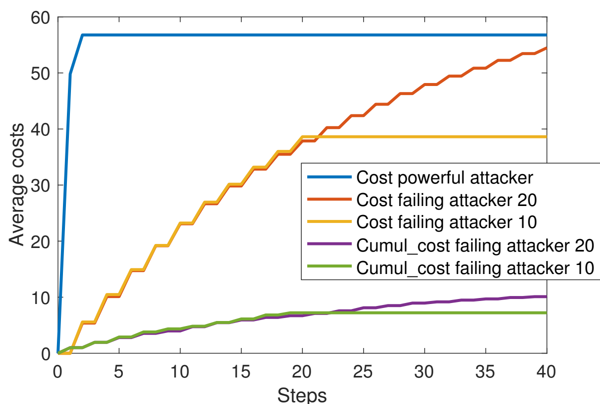


Fig. 10. Costs of successful attacks

ants (products) in a single, enriched LTS (family) model. None of these languages allow the specification of probabilistic SPL models; the few languages that do will be discussed in Section 7.2 when we discuss probabilistic SPL model-checking approaches.

Featured Transition Systems (FTSs) were introduced in [49] and further elaborated in [55], [61]. An FTS models a family of LTSs (one per product) that can be obtained by projecting on the feature expressions (Boolean formulae defined over the set of features) decorating the transitions: all transitions whose feature expression is not satisfied by the specific product's set of features are removed, as well as all states and transitions that have become unreachable. Adaptive FTSs, introduced in [61], allow the set of active features to vary dynamically, i.e. features can also be deactivated. QFLAN allows more general constraints than FTS' feature expressions (cf. Section 3.2) and adaptive or dynamic SPLs can be modeled as well; its action constraints are similar to the adaptation mechanisms of context-oriented programming as discussed and compared in [62].

Modal Transition Systems (MTSs) [63] were introduced to model successive refinements (implementations) of partial specifications. An MTS is an LTS distinguishing admissible (may) from necessary (must) transitions. In [64], MTSs were recognized as a suitable behavioral model for describing SPLs capable of checking the conformance of the behavior of a product against that of its product family. In a series of papers culminating in [7], MTSs were equipped with variability constraints which can express any Boolean function over the action labels, thus including all constraints that are typically defined by a variability model (but now expressed in terms of actions). Like FTSs, they model a family of LTSs (one per product) which can be obtained by turning each admissible but not necessary transition into a necessary transition or by removing it. Comparisons of FTSs and MTSs have appeared in the literature [48], [65]. Further MTS variants are variable I/O automata [66] and modal I/O automata [67]. QFLAN allows feature attributes and richer (quantitative) constraints (cf. Section 3.2) than any of these MTS models does, none of which moreover allows to model dynamic SPLs; the feature set is statically determined upfront.

Several process-algebraic theories for the modeling and analysis of SPLs have also been developed. In a series of papers, summarized in [68], Product Line CCS (PL-CCS) was defined as an extension of CCS by a variant operator allowing the user to model alternative behavior in the form of alternative processes, intending only one of them to exist at runtime. The choice calculus was introduced in [2] with the specific aim of providing a fundamental model for software variation, akin to the lambda calculus for programming languages. Another extension of CCS, DeltaCCS [8], was inspired by the well-known delta-modeling approach of automated product derivation for SPLs based on deltas that specify changes to be applied incrementally to a core product (cf. e.g. [69]). This modular approach differs from PL-CCS and the choice calculus, where choices are applied at well-defined variation points. Model-checking algorithms were implemented in MAUDE for SPLs specified in DeltaCCS against modal μ-calculus formulas. In [6], a so-called Variant Process Algebra (VPA) is defined to formally reason on SPLs. Like [64], it focuses on behavioral (bi)simulation relations instead of verification through model checking. Our process-algebraic FLAN family distinguishes itself from all these approaches by the explicit modeling it offers for a rich set of constraints that may concern quantitative aspects of feature attributes. While PL-CCS and DeltaCCS allow some minimal restructuring functionality, none of these process-algebraic approaches can model dynamic SPLs.

Other known formalisms, quite different from QFLAN, that were equipped with variability notions concern Petri nets [51], [70], Event-B [71], Finite State Machines [72] and UML Activity Diagrams [73], [74]. In the latter, performance properties are captured by annotations (such as the duration to execute an activity of an activity node) and interpreted as CTMCs. Combined with the above mentioned delta-modeling approach, these constitute the first attempts to efficient performance modeling of SPLs.

7.2 Model checking

We next describe a number of SPL model-checking tools that have been introduced for the above modeling languages, followed by an overview of probabilistic SPL model-checking approaches. We are not aware of any other work than ours on statistical model checking for SPLs.

We only discuss approaches that, like ours, analyze behavioral models. There are also numerous SPL analysis approaches that operate directly on the source code, often obtained by adapting existing tools for software model checking to deal with variability. Examples include an adaptation of PROMOVER [75] with variability annotations [76] for Java and the SPLVERIFIER [77] tool chain built on JAVA PATHFINDER [78] for Java and CPACHECKER [79] for C code. SPLVERIFIER uses standard off-the-shelf model-checking techniques to verify the absence of feature interactions by an approach called feature-aware verification. For further details and for other model-checking approaches in SPLE than the ones described next, we refer to the survey [5] which covers not only SPL model checking, but also type checking, static analysis and theorem proving and which distinguishes, besides product-based and family-based analyses, also feature-based analyses (which are not relevant to our approach given that features are only implicitly present as actions in our model).

We begin with several dedicated SPL model checkers.

The tool suite PROVELINES [80] supports discrete as well as real-time models, various types of computations, and advanced feature notions. All tool variants share the same common input language fPromela, which is an extension of the Promela input language of the well-known SPIN model checker (http://spinroot.com/). It includes the SPL model checker SNIP [81] for the verification of fLTL (feature LTL) properties over FTSs. A prototypical extension of the NuSMV model checker [82] uses a fully symbolic algorithm for the verification of fCTL (feature CTL) properties over FTSs specified in fSMV, which is a feature-oriented extension of the input language of (NU)SMV that was independently developed in the context of research on the renown problem of feature interaction [53]. In [20], SMT solving is implemented on top of SNIP (with Z3), i.e. behavioral models written in fPromela (with an FTS semantics) with additional arithmetic constraints. Admittedly, the resulting tool SNIP-Z3 does not scale well with the model size.

VMC (http://fmt.isti.cnr.it/vmc/) [83] is a tool for modeling and analyzing the behavior of SPLs modeled as MTSs with variability constraints [7]. Properties must be expressed in v-ACTL [84], a variability-aware action- and state-based branching-time temporal logic derived from the family of logics based on ACTL, the action-based version of CTL.

Next we turn to two off-the-shelf model checkers that were made amenable to SPL model checking, followed by a third one that offers probabilistic SPL model checking.

In [4], [85], a feature-oriented modular verification approach was developed, using an interpretation of FTSs in the MCRL2 formal specification language and toolset [86], [87]. MCRL2's parametrized data language allows one to handle feature attributes and quantitative constraints, like QFLAN. In order to perform family-based SPL model checking, a feature-oriented variant of the modal μ-calculus, with an FTS semantics, was introduced in [88] by incorporating feature expressions into the modal operators, thus generalizing the work on the feature-oriented variants fLTL and fCTL. In [10], it was shown how to exploit this logic for family-based model checking with MCRL2 as-is by encoding it back into the logic of MCRL2.

In [89], [90], it was shown how to use SPIN for family-based model checking of LTL formulas against FTSs by means of an additional automatic variability-specific abstraction refinement method based on the discovery of spurious counterexamples obtained during model checking.

PROFEAT, a software tool built on top of PRISM for the analysis of feature-aware probabilistic models is presented in [30]. It provides a guarded-command language for modeling families of probabilistic systems as well as an automatic translation of family models to the input language of PRISM (i.e. featureless models). It can deal with probabilistic DSPLs by offering dynamic feature switching (i.e. activation and deactivation of features at runtime) and with feature attributes. The tool is evaluated through a number of case studies, including (probabilistic) versions of the Elevator benchmark SPL (cf. Section 6.1). Due to the nature of the analysis, i.e. statistical vs. precise probabilistic analysis, we were able to handle significantly larger variants of the Elevator SPL, viz. up to 40 floors rather than 4.

We close this section with a few pointers to probabilistic model checking of SPLs.

In [26], a Maple-based implementation is applied to a small running example (the usual coffee machine), while an empirical evaluation is limited to randomly generated behavioral models.

In [27], Discrete time Markov chain families (DTMCFs) are introduced as a model to specify the probabilistic behavior of an SPL. Moreover, a probabilistic model checking algorithm to verify Probablistic CTL (PCTL) formulas is defined. A tool is said to be forthcoming.

In [29], Featured DTMCs (FDTMCs) are introduced to model the probability of a transition being executed in a product. Verification of dependability (i.e. the probability to reach a success state) is formulated in PCTL. Furthermore, three family-based model-checking techniques are defined to verify stochastic SPLs modeled as FDTMCs derived from sequence diagrams.

In [28], Markov Decision Processes (MDPs) are used to model dynamic SPLs (in particular allowing the activation and deactivation of features at runtime), i.e. LTSs whose transitions have guards that formalize feature-dependent behavior annotated with probabilities and costs to model stochastic phenomena and resource constraints.

8 CONCLUSIONS AND FUTURE WORK

We have presented QFLAN, a quantitative modeling and verification environment for highly (re)configurable systems, such as dynamic SPLs, including Eclipse-based tool support. QFLAN offers a high-level DSL in which to specify system configurations and their probabilistic behavior as well as advanced statistical analyses of properties expressed in MultiQuaTEx. The QFLAN tool's GUI offers designers editing support typical of modern integrated development environments (such as auto-completion, syntax and error highlighting, etc.) for writing QFLAN specifications as well as MultiQuaTEx expressions.

We have shown a novel application of our approach to risk analysis of a safe lock system from the security domain as well as to classical examples of analysing the configuration and behavior of highly (re)configurable systems from the SPLE literature. Arguably the most important anomaly

of feature models is the void feature model anomaly [21], i.e. when the root feature of the feature model cannot be selected thus forbidding the existence of any possible configuration. It is worth mentioning that QFLAN can actually be used to analyze the probability of this anomaly to occur, by verifying the probability of installing the root feature. The case studies have shown an analysis speedup for QFLAN, with respect to our earlier prototypical implementation, of more than three orders of magnitude.

We see a number of possible research directions for future work. First, we could provide additional QFLAN semantics for specific applications. For instance, a stochastic QFLAN semantics based on continuous time Markov chains to enable the analysis of time-related properties or an alternative QFLAN semantics based on FTSs to enable the use of the PROVELINES tool suite [80].

Concerning the tool support, we could automatize its distributed analysis features (cf. Section 5), develop ad-hoc tool variants for attack trees (cf. Section 6.2), and improve interoperability with other tools, for instance for importing feature models designed with FeatureIDE [91].

Finally, we could study the automatic synthesis of constraints starting from higher-level representations, possibly obtained by integrating our approach with the strategies synthesizer Uppaal Stratego [92].

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