

# Efficient Static Analysis and Verification of Featured Transition Systems

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- 1 Background
  - software product lines
  - variability modelling and analysis
- 2 Key idea and aim
- 3 Featured Transition Systems (FTSs)
  - definition and examples
  - ambiguous FTSs and disambiguation
- 4 Efficient static analysis of FTSs
  - algorithm and experiments
- 5 Family-based model checking on FTSs
  - branching-time properties with VMC
  - linear-time properties with SPIN
  - implemented in FTS4VMC
- 6 Conclusion and Outlook



- Configurable (software) system whose variants (products) differ by the provided **features**, i.e. the functionality that is relevant for an end-user

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Vehicle type

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Body type

Saloon

Touring

Convertible

Coupé

Gran Turismo

Sports Hatch

Roadster

Sports Activity Coupé

Sports Activity Vehicle

Number of seats

30 Vehicles **465 Model variants**

1  
2  
3



**BMW 1 Series 3-door Sports Hatch (34)**  
from £ 17,775.00



**BMW 1 Series 5-door Sports Hatch (39)**  
from £ 18,305.00



**BMW 2 Series Coupé (14)**  
from £ 24,265.00



**BMW 3 Series Saloon (56)**  
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**BMW 3 Series Touring (54)**  
from £ 24,865.00



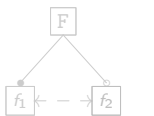
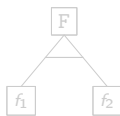
**BMW 3 Series Gran Turismo (39)**  
from £ 29,200.00

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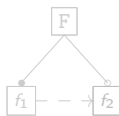


Benavides *et al.*, Automated Analysis of Feature Models 20 Years Later: a Literature Review. *Inf. Syst.*, 2010

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dead feature



false optional feature

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- Popular in embedded and critical systems domain: formal modelling and analysis techniques for proving SPL **behaviour** correct are widely studied  
Thüm *et al.*, A classification and survey of analysis strategies for SPLs. *ACM Comput. Surv.*, 2014
- Challenge existing formal methods and tools by potentially high number of different variants, each giving rise to a large state space, in general

33 optional, independent  
features



a unique configuration/variant for every

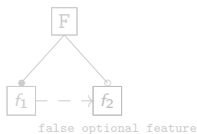
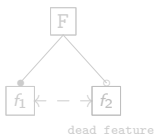
person on this planet

320<sup>optional, independent</sup> features

more configurations/variants than estimated

atoms in the universe

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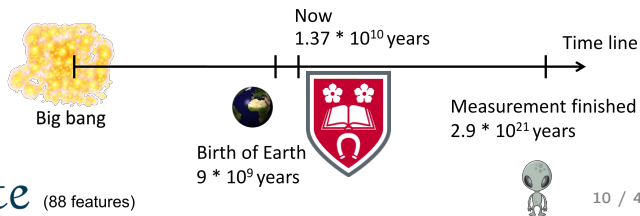
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⇒ Lift success stories known for single systems (products) to sets of products (families) by exploiting **variability** modelling and analysis






- 👍 Simple, brute-force approach
- 👍 Make use of standardly available, highly optimised analysis tools







- 👍 Simple, brute-force approach
- 👍 Make use of standardly available, highly optimised analysis tools
- 👎 Number of product variants is exponential in number of features
- 👎 Same piece of behaviour (or code) is verified numerous times, as many times as the number of variants that are able to execute it









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Dimovski et al., Family-based model checking without a family-based model checker @ SPIN'15

Chrzon et al., Family-based modeling and analysis for probabilistic systems: featuring ProFeat @ FASE'16

ter Beek et al., Family-Based Model Checking with mCRL2 @ FASE'17

Dimovski et al., Variability-specific Abstraction Refinement for Family-based Model Checking @ FASE'17

Dimovski, Abstract Family-Based Model Checking Using Modal Featured Transition Systems @ FASE'18

ter Beek et al., Family-Based SPL Model Checking Using Parity Games with Variability @ FASE'20

...



Mimick anomaly detection known from feature model analysis in behavioural SPL models (FTSs) by automated static analysis:

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2. false optional transitions
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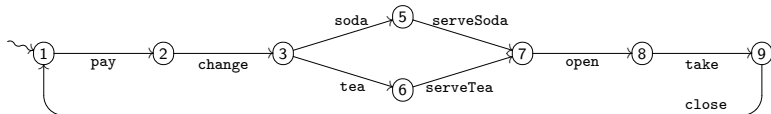
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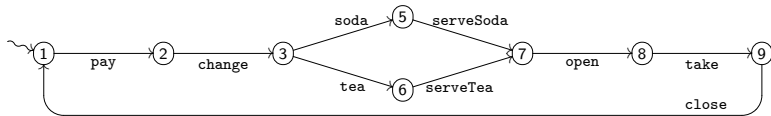
Catch and offer means to remove possible ambiguities in FTSs:

1. Ambiguous FTSs are undesired: they give unclear ideas of the SPLs
2. Unambiguous FTSs pave way to efficient family-based verification

A **Labelled Transition System** (LTS) is a quadruple  $(S, \Sigma, s_0, \delta)$  with states  $S$ , actions  $\Sigma$ , initial state  $s_0$ , and transitions  $\delta \subseteq S \times \Sigma \times S$



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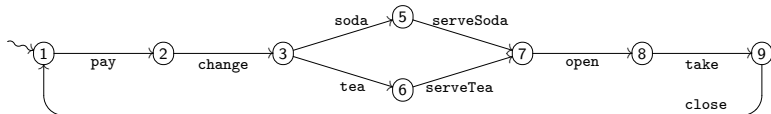


An FTS adds to this a feature model and feature expressions:

Classen et al., Model checking lots of systems @ ICSE'10, FTSs. *IEEE TSE*, 2013

A **Featured Transition System** (FTS) is a sextuple  $(S, \Sigma, s_0, \delta, F, \Lambda)$  with states  $S$ , actions  $\Sigma$ , initial state  $s_0$ , (**featured**) transitions  $\delta \subseteq S \times \Sigma \times \mathbb{B}(F) \times S$ , with Boolean (**feature**) expressions  $\mathbb{B}(F)$  over **features**  $F$ , and (**product**) configurations  $\Lambda \subseteq \{ \lambda : F \rightarrow \mathbb{B} \}$

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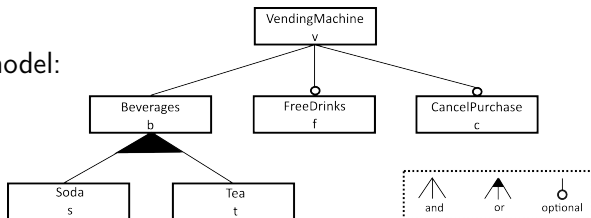
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LTS  $\mathcal{F}|_\lambda$  specified by configuration  $\lambda \in \Lambda$  is called a **product** of  $\mathcal{F}$  [remove: 1) all featured transitions whose feature expressions are not satisfied by  $\lambda$ ; 2) all unreachable states and their outgoing transitions]

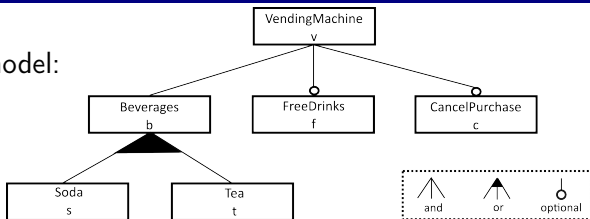
Feature model:



12 valid products

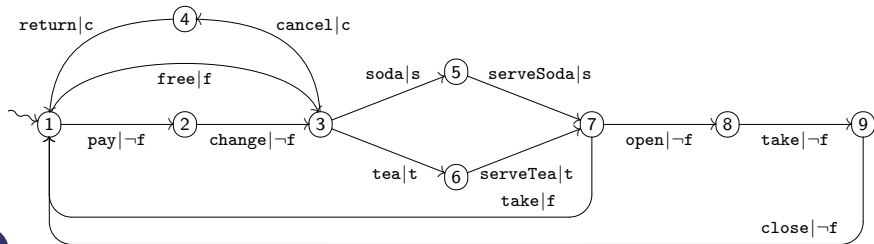
e.g., {v,b,s,t}, {v,b,s,c}

Feature model:

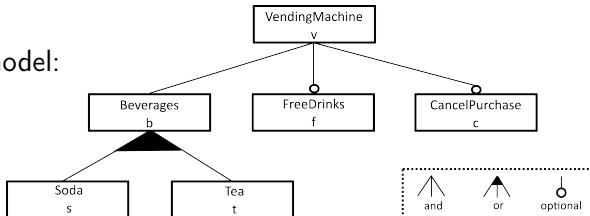


FTS of 12 valid products (LTSs)

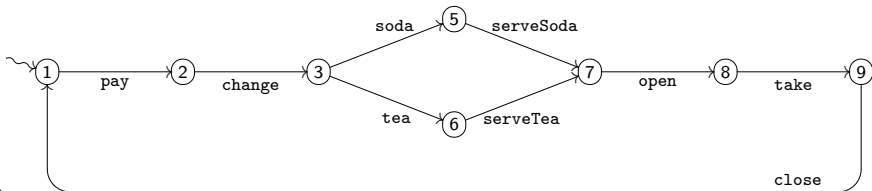
e.g.,  $\{v, b, s, t\}$ ,  $\{v, b, s, c\}$



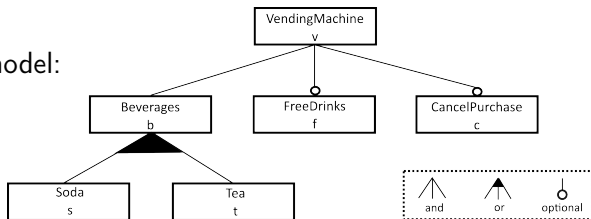
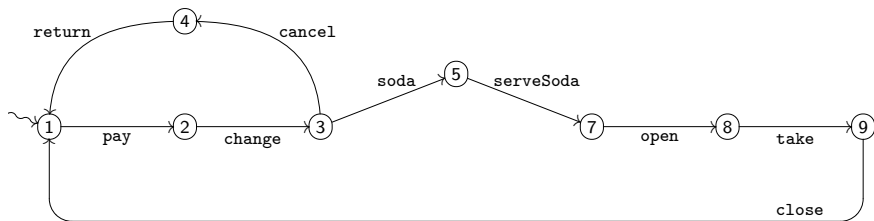
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dead transition an FTS transition not reachable in any product (LTS)

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**false optional transition** a featured FTS transition which is

1. not dead
2. not annotated with the feature expression  $\top$  (true, i.e., selected)
3. present in every FTS product in which its source state is present

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**hidden deadlock state** an FTS state which is

1. not a deadlock (i.e., it has outgoing transitions) in the FTS
2. a deadlock (i.e., no outgoing transitions) in some FTS product(s)

Transformation:

1. remove dead transitions

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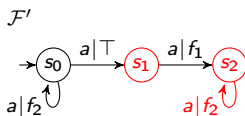
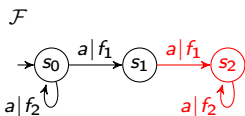
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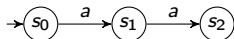
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  - 3.2  $\forall s$ : add a *deadlock transition* from  $s$  to  $s_{\dagger}$  labelled by  $\dagger \notin \Sigma$  and by a feature expression that negates the disjunction of the feature expressions of all outgoing transitions of  $s$

## Feature Model: $f_1 \oplus f_2$



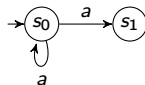
$$\mathcal{F}|_{\lambda_1} = \mathcal{F}'|_{\lambda_1}$$



$$\mathcal{F}|_{\lambda_2}$$

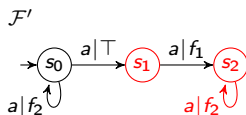
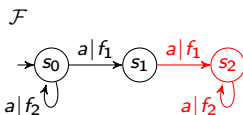


$$\mathcal{F}'|_{\lambda_2}$$

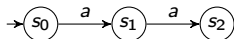


products  $\lambda_1 = \{f_1\}$  and  $\lambda_2 = \{f_2\}$

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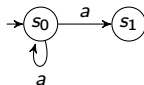
$$\mathcal{F}|_{\lambda_1} = \mathcal{F}'|_{\lambda_1}$$



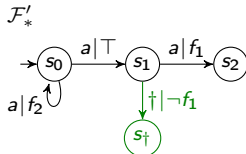
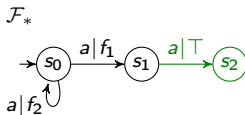
$$\mathcal{F}|_{\lambda_2}$$



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```
# fts contains the input FTS
for s in fts.states:
  if(s.out_trs = ∅):
    s.hdead ← False
  else:
    s.hdead ← check(exist_deadlock(s))
# for all states s, it holds that:
# (s.hdead ≡ "s is a hidden deadlock")
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# fts contains the input FTS
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for s in fts.states:
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```
  if(s.out_trs =  $\emptyset$ ):
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for s in fts.states:
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```
  for t in s.in_trs:
```

```
    t.dead  $\leftarrow$  not check(is_not_dead_transition(t))
```

```
    if(t.dead or t.bx =  $\top$ ):
```

```
      t.false_opt  $\leftarrow$  False
```

```
    else:
```

```
      t.false_opt  $\leftarrow$  not check(may_be_opt_transition(t))
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```
# for all transitions t, it holds that: (t.dead  $\equiv$  "t is dead")
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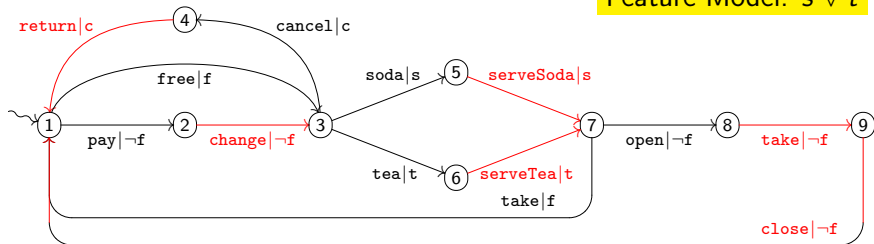
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- We prove its correctness
- Our implementation uses Z3



Feature Model:  $s \vee t$ 

## Result of static analysis on FTS

Vending Machine: live

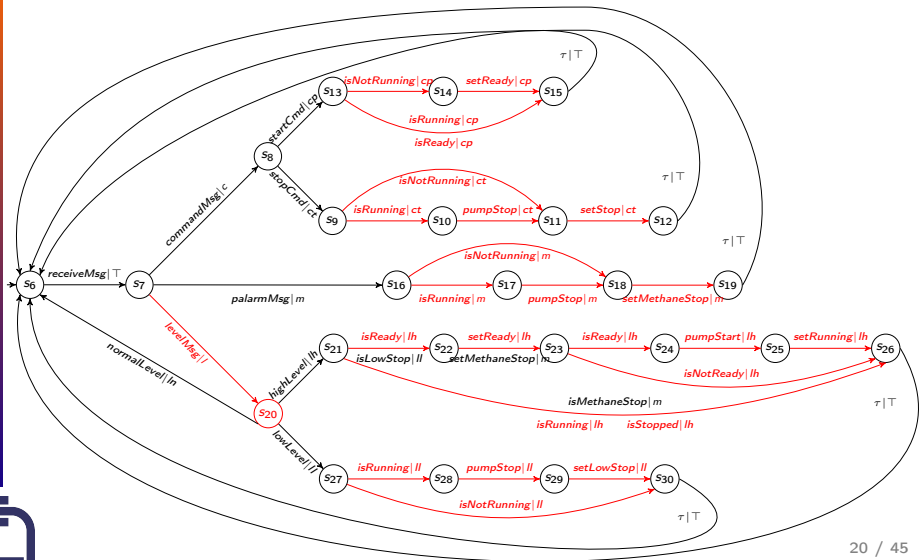
LIVE STATES = [1,2,3,4,5,6,7,8,9]

DEAD TRANSITIONS = []

FALSE OPTIONAL TRANSITIONS = [(2,3), (4,1), (5,7), (6,7), (8,9), (9,1)]

HIDDEN DEADLOCK STATES = []

Feature Model:  $(c \leftrightarrow (ct \vee cp)) \wedge l$



## Result of static analysis on FTS

Mine Pump: not live

LIVE STATES = [S6,S7,S8,S9,S10,S11,S12,S13,S14,S15,S16,S17,S18,  
S19,S21,S22,S23,S24,S25,S26,S27,S28,S29,S30]

DEAD TRANSITIONS = []

FALSE OPTIONAL TRANSITIONS = [(S10,S11), (S11,S12), (S13,S14),  
(S13,S15,isReady), (S13,S15,isRunning), (S14,S15), (S16,S17),  
(S16,S18), (S17,S18), (S18,S19), (S21,S22,isReady),  
(S21,S26,isRunning), (S21,S26,isStopped), (S22,S23,setReady),  
(S23,S24), (S23,S26), (S24,S25), (S25,S26), (S27,S28), (S27,S30),  
(S28,S29), (S29,S30), (S7,S20), (S9,S10), (S9,S11)]

HIDDEN DEADLOCK STATES = [S20]

FTS	characteristics			results of static analysis				computational effort	
	S	\delta	\Sigma	live-ness	# dead transitions	# false optional transitions	# hidden deadlock states	run-time (s)	memory usage (Mb)
Vending machine <small>Classen, PhD thesis, 2011</small>	9	13	12	yes	0	6	0	0.26	29.765
Coffee machine <small>Asirelli et al. @ SPLC'11</small>	14	23	15	yes	0	14	0	0.29	30.305
Soup component <small>Belder et al. @ FMSPL'15</small>	13	28	18	yes	0	7	0	0.316	30.85
Mine pump (system) <small>Classen, PhD thesis, 2011</small>	25	41	22	no	0	25	1	0.344	31.704
Mine pump (controller) <small>Classen, PhD thesis, 2011</small>	77	104	22	no	0	59	4	0.548	36.295
Coffee/Soup machine <small>Belder et al. @ FMSPL'15</small>	182	691	33	yes	8	284	0	37.766	119.427
Mine pump (complete) <small>Classen, PhD thesis, 2011</small>	418	1,255	26	yes	0	308	0	98.994	119.127
Claroline <small>Devroye et al. @ VaMoS'14</small>	107	11,236	106	yes	0	259	0	2413.8	2010.229

FTS	characteristics			computational effort				results
				implementation [SPLC19]		implementation [EMSE21]		
	Model	S	\delta	\Sigma	runtime (s)	memory usage (Mb)	runtime (s)	memory usage (Mb)
Vending machine <small>Classen, PhD thesis, 2011</small>	9	13	12	0.92	38.230	0.26	29.765	3.54x
Coffee machine <small>Asirelli et al. @ SPLC'11</small>	14	23	15	2.822	40.140	0.29	30.305	9.72x
Soup component <small>Belder et al. @ FMSPLE'15</small>	13	28	18	2.544	40.870	0.316	30.85	8.05x
Mine pump (system) <small>Classen, PhD thesis, 2011</small>	25	41	22	2.192	41.899	0.344	31.704	6.37x
Mine pump (controller) <small>Classen, PhD thesis, 2011</small>	77	104	22	8.12	49.091	0.548	36.295	14.82x
Coffee/Soup machine <small>Belder et al. @ FMSPLE'15</small>	182	691	33	timeout	-	37.766	119.427	>7200.00x
Mine pump (complete) <small>Classen, PhD thesis, 2011</small>	418	1,255	26	timeout	-	98.994	119.127	>7200.00x
Claroline <small>Devroey et al. @ VaMoS'14</small>	107	11,236	106	timeout	-	2413.8	2010.229	>7200.00x

FTS	characteristics			computational effort				results
	S	$\delta$	$\Sigma$	full-fledged implementation		specialised implementation		runtime fraction
				runtime (s)	memory usage (Mb)	runtime (s)	memory usage (Mb)	
Coffee/Soup machine <small>Belder et al. @ FMSPL'15</small>	182	691	33	37.766	119.427	2.288	61.620	6.06%
Mine pump (complete) <small>Classen, PhD thesis, 2011</small>	417	1,255	26	98.994	119.127	2.948	68.969	2.97%
Claroline <small>Devroey et al. @ VaMoS'14</small>	107	11,236	106	2413.8	2010.229	86.752	551.888	3.59%

## Main ingredient: Modal Transition System (MTS)

- LTS distinguishing admissible (**may**) and necessary (**must**) transitions

Larsen & Thomsen, A Modal Process Logic @ LICS'88

- Recognised as a useful formalism to describe in a **compact** way the possible **behaviour** of all the products (LTSs) of a product family

Fischbein *et al.*, A foundation for behavioural conformance in SPL architectures @ ROSATEA'06

Fantechi & Gnesi, A behavioural model for product families @ ESEC/FSE'07

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Larsen *et al.*, Modal I/O Automata for Interface and Product Line Theories @ ESOP'07

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- 👍 Our solution: add a set of **variability constraints** to the MTSs to be able to decide which derivable products (LTSs) are valid ones

Asirelli *et al.*, Formal Description of Variability in Product Families @ SPLC'11

ter Beek *et al.*, Modelling and analysing variability in product families. *JLAMP*, 2016



Recall:

A **Labelled Transition System** (LTS) is a quadruple  $(Q, \Sigma, s_0, \rightarrow)$  with states  $S$ , actions  $A$ , initial state  $s_0$ , and transitions  $\rightarrow \subseteq S \times \Sigma \times S$

Next we define MTSs with variability constraints:

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A **Labelled Transition System** (LTS) is a quadruple  $(Q, \Sigma, s_0, \rightarrow)$  with states  $S$ , actions  $A$ , initial state  $s_0$ , and transitions  $\rightarrow \subseteq S \times \Sigma \times S$

Next we define MTSs with variability constraints:

A **Modal Transition System** (MTS) is a quintuple  $(S, \Sigma, s_0, \rightarrow_{\square}, \rightarrow_{\diamond})$  such that  $(S, \Sigma, s_0, \rightarrow_{\square} \cup \rightarrow_{\diamond})$  is an LTS, called its underlying LTS

An MTS has two distinct transition relations

1. **may** transition relation  $\rightarrow_{\diamond} \subseteq S \times \Sigma \times S$ : **admissible** transitions
2. **must** transition relation  $\rightarrow_{\square} \subseteq S \times \Sigma \times S$ : **necessary** transitions

By definition, any necessary transition is also admissible:  $\rightarrow_{\square} \subseteq \rightarrow_{\diamond}$   
 (denote  $\dashrightarrow \equiv \rightarrow_{\diamond} \setminus \rightarrow_{\square}$ : **optional** transitions)

Variability constraints of form **AL**Ternative, **EX**Cludes, **RE**QUIRES, etc.

$a_1$  **ALT**  $\cdots$  **ALT**  $a_n$  : precisely one among the  $n \geq 2$  actions  $a_1, \dots, a_n$  is reachable in  $\mathcal{L}$  (i.e. is the label of a reachable transition)

$b_1$  **OR**  $\cdots$  **OR**  $b_n$ , where  $b_i$  is either  $a_i$  or  $\neg a_i$  : at least one among the conditions on  $n \geq 2$  actions  $b_1, \dots, b_n$  holds, i.e.  $b_i = a_i$  is reachable in  $\mathcal{L}$  or  $b_i = \neg a_i$  is not reachable in  $\mathcal{L}$

$a_1$  **EXC**  $a_2$  : at most one of the actions  $a_1$  and  $a_2$  is reachable in  $\mathcal{L}$

$a_1$  **REQ**  $a_2$  : action  $a_2$  is reachable in  $\mathcal{L}$  whenever  $a_1$  is reachable in  $\mathcal{L}$

$a_1$  **REQ** ( $a_2$  **ALT**  $\cdots$  **ALT**  $a_n$ ) : precisely one among the  $n \geq 2$  actions  $a_2, \dots, a_n$  is reachable in  $\mathcal{L}$  if  $a_1$  is reachable in  $\mathcal{L}$

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A **product LTS** is obtained from a family  $MTS_v$  in the following way:

1. include **all** (reachable) must transitions and
2. include **subset** of the (reachable) optional transitions, remove rest
3. satisfy assumptions of **coherence** and **consistency**
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The set  $\{\mathcal{P}_i = (S_i, \Sigma, s_0, \delta_i) \mid i > 0\}$  of **product LTSs** is obtained by considering each pair of  $S_i \subseteq S$  and  $\delta_i \subseteq \delta^\diamond \cup \delta^\square$  to be defined s.t.

1. every  $s \in S_i$  is reachable in  $\mathcal{P}_i$  from  $s_0$  via transitions from  $\delta_i$
2. there exists no  $(s, a, s') \in \delta^\square \setminus \delta_i$  such that  $s \in S_i$
3. LTS is consistent: both  $\xrightarrow{-a} \rightsquigarrow \xrightarrow{a}$  and  ~~$\xrightarrow{a}$~~  not allowed
4.  $\mathcal{P}_i$  satisfies all variability constraints in  $\Upsilon$

**Theorem (FTS2MTS<sub>v</sub> transformation is sound and complete)**

*Let  $\mathcal{F}$  be an FTS and let  $\mathcal{M}$  be the MTS<sub>v</sub> generated from  $\mathcal{F}$  according to the FTS2MTS<sub>v</sub> model transformation algorithm. Then the sets of derived variants  $\text{lts}(\mathcal{F})$  and  $\text{lts}(\mathcal{M})$  coincide, up to dummy transitions and action relabelling.*

ter Beek et al., From FTSs to MTSs with Variability Constraints @ SEFM'15

### Theorem (FTS<sub>2</sub>MTS<sub>v</sub> transformation is sound and complete)

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ter Beek et al., On the Expressiveness of MTSs with Variability Constraints. SCP, 2019

VMC profits from optimisations of KandISTI family of model checkers: bounded, on-the-fly model checking with **linear complexity**, providing clear and easily understandable **explanations** when the validation fails

ter Beek *et al.*, A state/event-based model-checking approach for the analysis of abstract system properties  
© SCP, 2011

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VMC accepts as input a specification in (value-passing) modal process algebra, possibly with additional variability constraints:

- interactively explore the model ( $MTS_v$ )
- derive and explore (all) the model's valid products (LTSs)
- visualise the model/products graphically as  $MTS_v$ /LTSs
- verify branching-time v-ACTL properties over  $MTS_v$ /LTSs
- interactively explain why a property is (not) satisfied

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VMC is freely usable online:

<http://fmt.isti.cnr.it/vmc/>

VMC notifies whenever preservation of an analysis result is applicable

**VMC V6.5**  
(2019)

● ● ● ● ●

Edit Model

---

View Current Model

Explore the MTS

Draw Family MTS


---

Generate Products

---

Welcome

Quit



Kandinsky 1908

**The Formula:** `not E[true {not tea} U {serveTea}true]`  
**is TRUE**

The formula holds for ALL the MTS variants

(evaluation time= 0.067 sec.)

---

(total states generated= 9, computations fragments generated= 17, total evaluation time= 0.067 sec.)

---

**ACTL-UCTL-SocL-vACTL**

not E [true {not tea} U {serveTea} true]

Check  
The  
Formula

Explain  
the  
Result

*It is not possible that serveTea occurs without being preceded by tea*

VMC lists for each product the action labels of all may transitions that have been preserved (as must transitions) in that product

**VMC V6.5**  
(2019)

● ● ● ● ●

New Model ...

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---



Kandinsky 1908

**Evaluation of the formula "[pay] AF {takePaid} true" on all family products**

<a href="#">product+CancelPurchase+FreeDrinks+Soda+Tea_12</a>	Formula evaluates	TRUE
<a href="#">product+CancelPurchase+FreeDrinks+Soda_08</a>	Formula evaluates	TRUE
<a href="#">product+CancelPurchase+FreeDrinks+Tea_10</a>	Formula evaluates	TRUE
<a href="#">product+CancelPurchase+Soda+Tea_06</a>	Formula evaluates	FALSE
<a href="#">product+CancelPurchase+Soda_02</a>	Formula evaluates	FALSE
<a href="#">product+CancelPurchase+Tea_04</a>	Formula evaluates	FALSE
<a href="#">product+FreeDrinks+Soda+Tea_11</a>	Formula evaluates	TRUE
<a href="#">product+FreeDrinks+Soda_07</a>	Formula evaluates	TRUE
<a href="#">product+FreeDrinks+Tea_09</a>	Formula evaluates	TRUE
<a href="#">product+Soda+Tea_05</a>	Formula evaluates	TRUE
<a href="#">product+Soda_01</a>	Formula evaluates	TRUE
<a href="#">product+Tea_03</a>	Formula evaluates	TRUE

**Logic Formula for all Products**

[pay] AF {takePaid} true

Check The Formula	Explain the Result
-------------------------	--------------------------

*Whenever pay occurs, eventually take(Paid) occurs*

## Note

1. Any FTS  $\mathcal{F}$  can trivially be transformed into an MTS  $\mathcal{F}_{\text{MTS}}$  (must  $\rightarrow$  necessary transitions, featured  $\rightarrow$  optional transitions, all transitions admissible, remove all feature expressions)
2. If the FTS is unambiguous, then the corresponding MTS is **live** (as it has no hidden deadlocks, and all must transitions are necessary)

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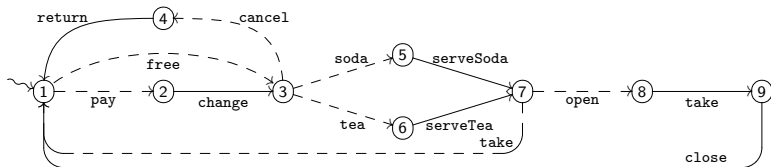
ter Beek et al., Modelling and analysing variability in product families. *JLAMP*, 2016

Any formula  $\phi$  of  $v\text{-ACTLive}^\square$  is preserved by live FTSs: given a live FTS  $\mathcal{F}$ , whenever  $\mathcal{F}_{\text{MTS}} \models \phi$ , then  $\mathcal{F}|_\lambda \models \phi$  for all products  $\mathcal{F}|_\lambda$  of  $\mathcal{F}$

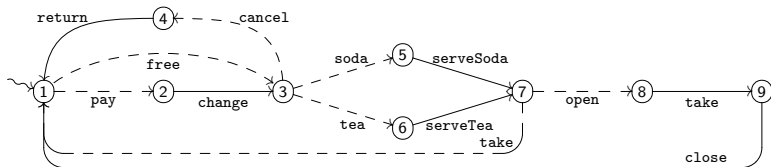


Example v-ACTLive<sup>□</sup> formulae that could now be verified with VMC:  
(with a **linear complexity**)

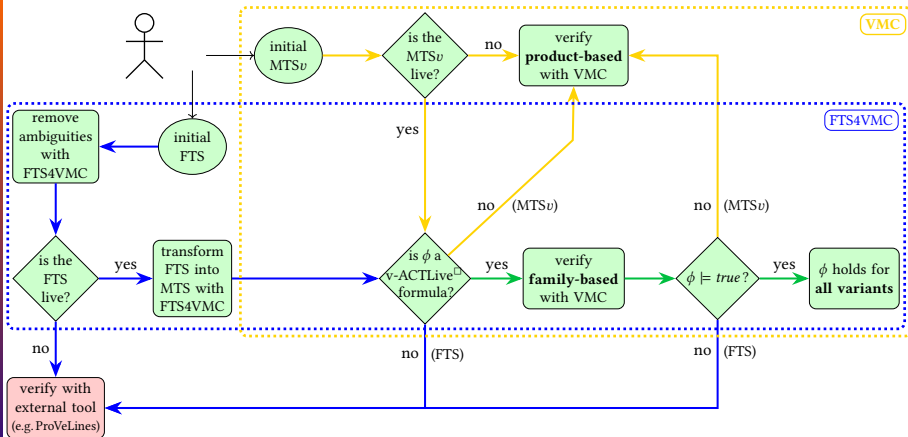
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(with a **linear complexity**)



1.  $AG AF_{pay \vee free} \top$ : infinitely often, either action *pay* or action *free* is executed
2.  $AG [open] AF_{close} \top$ : it is always the case that the execution of action *open* is eventually followed by that of action *close*
3.  $AG AF_{cancel \vee serveSoda \vee serveTea} \top$ : infinitely often, either action *cancel* or action *serveSoda* or action *serveTea* is executed
4.  $\neg E [\top \neg_{tea} U_{serveTea} \top]$ : it is not possible that action *serveTea* is executed without being preceded by an execution of action *tea*
5.  $[pay] AF_{take \vee cancel} \top$ : whenever action *pay* is executed, eventually also either action *take* or action *cancel* is executed



The **green** blocks are automated by the toolchain (FTS4VMC+VMC)

The **blue** and **green** steps (applied to FTSs) are realised by FTS4VMC

Select FTS Model

Upload model

Delete model and close

Download displayed result

Full ambiguities analysis

Liveness analysis

Stop processing

Remove all ambiguities

Remove false optional transitions

Remove dead transitions and hidden deadlock states

Apply transformation

View modal transition system

Verify property

Insert an ACTL formula to verify

Show explanation

Legend

dead transition

false optional transition

hidden deadlock

### Analysis of vendingnew.dot

FTS

Console Source Graph Summary Counterexample graph

```

graph LR
    1((1)) -- "take | f" --> 4((4))
    1 -- "return | c" --> 4
    1 -- "free | f" --> 2((2))
    1 -- "cancel | c" --> 4
    1 -- "puy | not f" --> 2
    1 -- "change | not f" --> 3((3))
    1 -- "soda | s" --> 5((5))
    1 -- "tea | t" --> 6((6))
    1 -- "close | not f" --> 7((7))
    2 --> 3
    3 --> 5
    3 --> 6
    5 -- "serveSoda | s" --> 7
    6 -- "serveTea | t" --> 7
    7 -- "open | not f" --> 8((8))
    8 -- "take | not f" --> 9((9))
  
```

The screenshot displays the FTS4VMC web interface. On the left is a vertical sidebar with green buttons for various actions: Select FTS Model, Upload model, Delete model and close, Download displayed result, Full ambiguities analysis, Liveness analysis, Stop processing, Remove all ambiguities, Remove false optional transitions, Remove dead transitions and hidden deadlock states, Apply transformation, View modal transition system, and Verify property. Below the 'Verify property' button is a text input field for inserting an ACTL formula and a 'Show explanation' button at the bottom.

**Analysis of vendingnew.dot**

**FTS**

Console Source Graph Summary Counterexample graph

Number of states: 9

Number of transitions: 13

**Ambiguities found**

**Dead transitions**

**False optional transitions**

- (2, 3): change | Not(f)
- (4, 1): return | c
- (5, 7): serveSoda | s
- (6, 7): serveTea | t
- (8, 9): take | Not(f)
- (9, 1): close | Not(f)

**Hidden deadlock states**

Select FTS Model

Upload model

Delete model and close

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    3 -- tea --> 6((6))
    4 -- return --> 1
    5 -- serveSoda --> 7
    6 -- serveTea --> 7
    7 -- take --> 1
    7 -- open --> 8((8))
    8 -- take --> 9((9))
    9 -- close --> 1
  
```



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Model checking LTL formulae on an LTS reduces to analysing its maximal paths: an LTL formula is valid if it holds for all maximal paths

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Note

1. Trivially carries over to FTSs by ignoring the feature expressions
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Thus:

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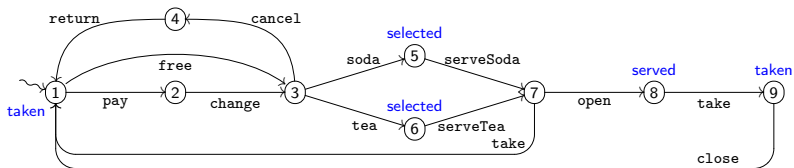


Example LTL formulae that could now be verified with SPIN (for instance):

<http://spinroot.com/>

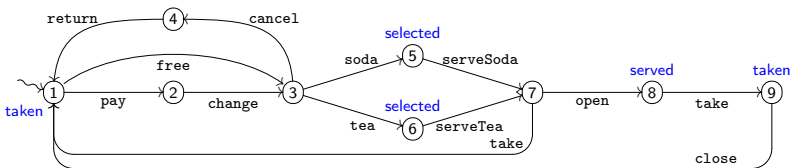
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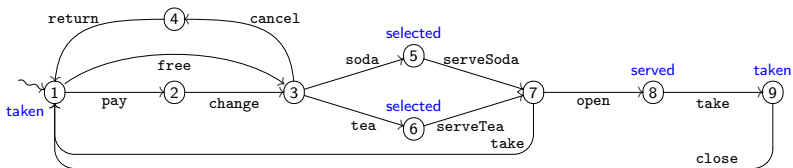
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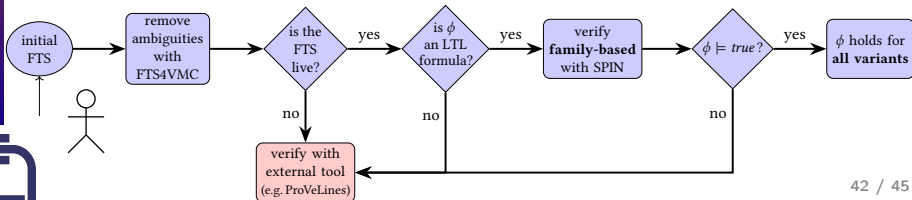
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- benchmark experiments

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  - both linear- and branching-time properties
3. Automated by a toolchain:
  - front-end tool FTS4VMC
4. Future work:
  - integrate FTS2PROMELA transformation

- SPLC19 M.H. ter Beek, F. Damiani, M. Lienhardt, F. Mazzanti, and L. Paolini, Static Analysis of Featured Transition Systems. In Proceedings of the 23rd International Systems and Software Product Line Conference (SPLC'19), Paris, France, ACM Press, New York, 2019, 39–51 ([best paper](#))
- EMSE21 M.H. ter Beek, F. Damiani, M. Lienhardt, F. Mazzanti, and L. Paolini, Efficient Static Analysis and Verification of Featured Transition Systems. *Empirical Software Engineering* (2021) ([to appear](#))
- SPLC21 M.H. ter Beek, F. Damiani, F. Mazzanti, G. Scarso, and M. Valfrè, Static Analysis and Family-based Model Checking of Featured Transition Systems with VMC. Zenodo (February 2021) ([to be submitted](#))  
<http://doi.org/10.5281/zenodo.4497888>

Hope to see you all at SPLC 2021 'here' in Leicester:

25th Systems and Software Product Line Conference

**Submission deadline research papers: April 23, 2021**

PC chairs:

- Maurice H. ter Beek, ISTI-CNR, Pisa, Italy
- Ina Schaefer, TU Braunschweig, Germany

Many more tracks (industry, challenges, demos & tools, journal first, doctoral symposium, hall of fame) plus workshops and tutorials

Organised by Mohammad Mousavi and his team