

*Quantitative Modelling and Analysis of Variability*  
~~An Introduction to Software Product Lines and~~  
~~Modelling and Analysing Variability~~  
*in Highly Configurable Systems*

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Based on joint work with Stefania Gnesi, Franco Mazzanti (ISTI-CNR),  
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~~IPA Fall Days on Models in Software Engineering~~  
~~WestCord de Veluwe, Garderen, The Netherlands~~  
~~October 30, 2019~~ *PhD in Smart Computing*

*July 13, 2021*

# Outline

- 1 Introduction to Software Product Lines
- 2 Modelling Variability
  - Featured Transition Systems (fPROMELA, fSMV, fLTL, fCTL)
  - Modal Transition Systems (with variability constraints, v-CTL)
  - Flan, PFLan, QFLan, ProFeat
- 3 Analysing Variability
  - SNIP, ProVeLines, fNuSMV, SPIN, mCRL2
  - VMC
  - QFLan, ProFeat, PRISM
- 4 Outlook

# An introduction to Software Product Lines

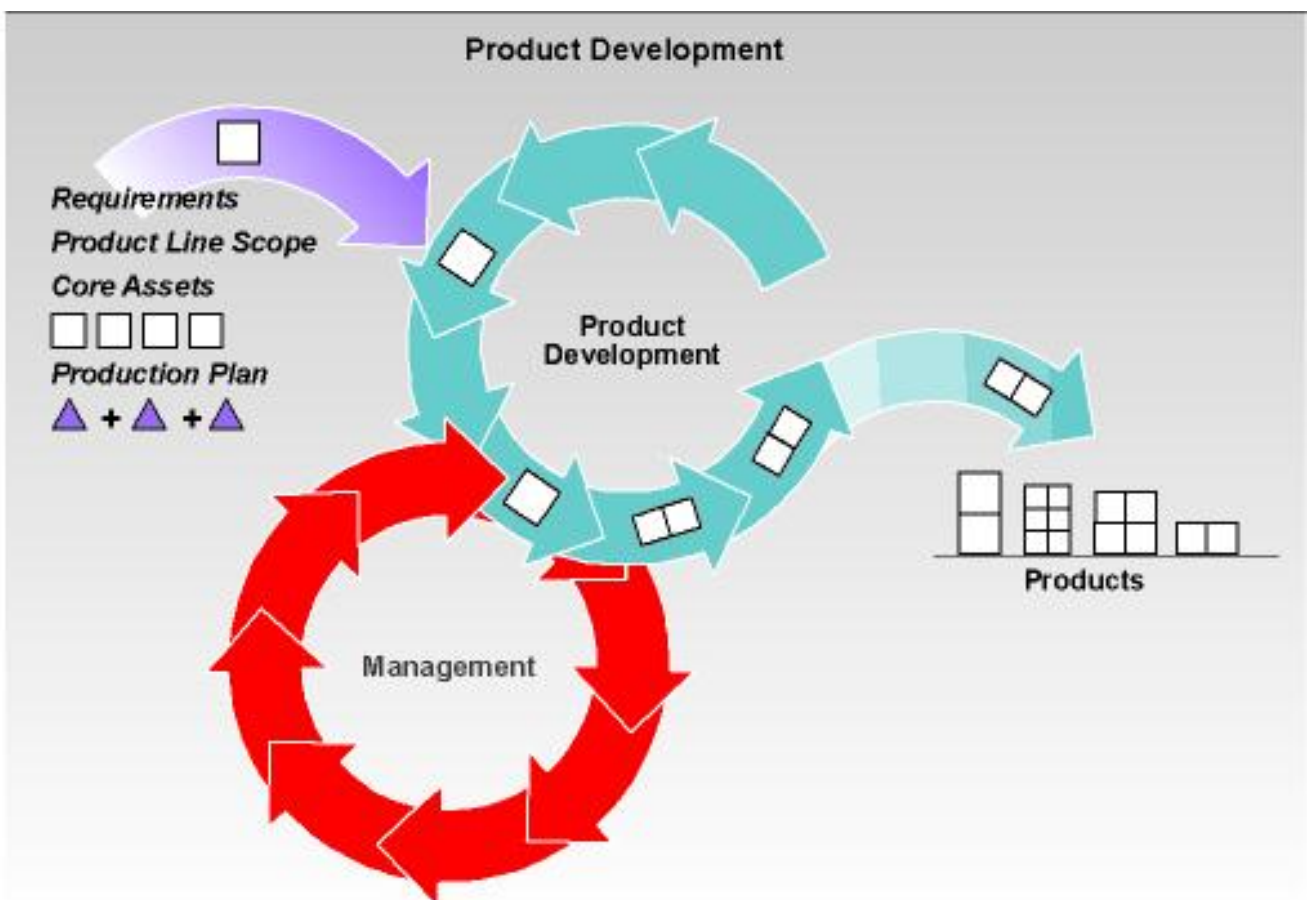
- (Software) Product Line Engineering
- Product variability and features
- Variability modelling and analysis

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*These slides are partly based on slides by Klaus Pohl, Sven Apel and Slinger Jansen*

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# Product line engineering in theory...



# ...and in practice



# Software Product Line Engineering

- Developing a family of products from a **common reference model** (usually in terms of features) and **mass customisation** (to serve different markets)

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- Two distinct development processes
  
- Variability

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- Two distinct development processes
  - **Domain engineering**: develop reusable domain artefacts
  - **Application engineering**: develop individual products by reusing domain artefacts
- Variability
  - **Common features** that are part of all products
  - **Variable features** that can be selected for a product

# Variability in requirements

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R12: The navigation system must allow the user to make inputs using a control panel or by voice entry

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  1. A navigation system that allows the user to make inputs only via the control panel
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  3. A navigation system that allows the user to make inputs only via the control panel and by voice entry
- Conjunction is a logical “or” or an exclusive “or”?
- Is only one system asked for, or two or three different systems?

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R12: “input via control panel”, “input via voice entry”

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- **Domain engineering**: explicit documentation of variability supports the identification of possible variable aspects and **fosters explicit decisions** about which aspects shall be variable in the product family (variation points) and which options shall exist for each variable aspect (variants);

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- **Application engineering**: explicit documentation of variability in terms of variation points and variants **supports system development** by making explicit the necessary decisions and decision options

# Features

- What is a feature?

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- What is a feature?
  - End-user visible behaviour or property of a system...
  - ...that may be optional and/or may have alternatives
- Features represent **commonalities and variabilities** of (software) systems



# What's in a *feature*?

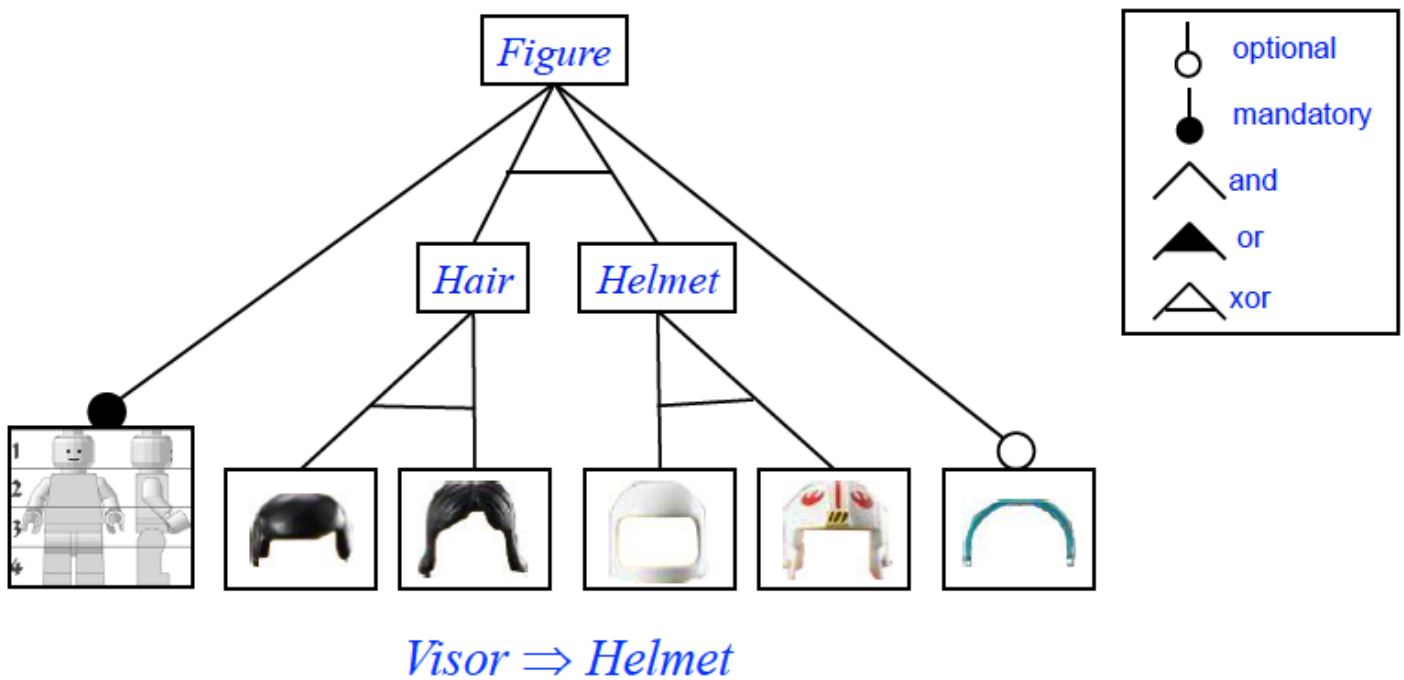
Reference	Definition
Kang <i>et al.</i> [3]	"a prominent or distinctive user-visible aspect, quality or characteristic of a software system or systems"
Kang <i>et al.</i> [8]	"distinctively identifiable functional abstractions that must be implemented, tested, delivered, and maintained"
Eisenecker and Czarnecki [6]	"anything users or client programs might want to control about a concept"
Bosch <i>et al.</i> [9]	"A logical unit of behaviour specified by a set of functional and non-functional requirements."
Chen <i>et al.</i> [10]	"a product characteristic from user or customer views, which essentially consists of a cohesive set of individual requirements"
Batory [11]	"an elaboration or augmentation of an entity(s) that introduces a new service, capability or relationship"
Batory <i>et al.</i> [12]	"an increment in product functionality"
Apel <i>et al.</i> [13]	"a structure that extends and modifies the structure of a given program in order to satisfy a stakeholder's requirement, to implement and encapsulate a design decision, and to offer a configuration option."



(Classen *et al.* @ FASE'08)

# Feature selection

Typically, only a subset of feature combinations is **valid**



# Products and product lines

- Product  $\Leftrightarrow$  valid feature combination (configuration)





# Features and software

- Main concept in SPLE
  - **Easy to use** in informal models
  - Easily converts into business: **product sales**
  - Easily converts into product design: **variability**
  - **Enables reuse** of features

# Features and software

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  - **Enables reuse** of features
- In telecommunication, features became popular in the 1960s with the advent of computer-controlled telephone switches; telecommunication software has been conceived in terms of features ever since
  - **1992-2009** International Conference on Feature Interactions in Telecommunications and Software Systems

# Variability models: explicit decisions

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# Variability models: explicit decisions

- Compact representations of all products of a product family in terms of their features
- **Feature diagram/model**: hierarchical tree structure, with the family as its root, features as its nodes and possibly further cross-tree constraints (Kang *et al.* '90)
- **Common Variability Language (CVL)**: OMG's effort to standardise variability modelling as a separate and generic language to define variability on base models
- **Orthogonal Variability Model** (Pohl *et al.* '05), etc., etc.

## Advantages of dedicated conceptual model

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- **Improved communication** with different stakeholders (e.g. communicate to customers which variants can be selected at which variation points)
- **Transparent decisions** i.e. the originator of a variation point is forced to state the rationale for introducing variability in a specific domain artefact
- Relationships between requirements and variants become **traceable** (e.g. stakeholders can document which requirements, design, implementation and test artefacts are influenced by a variant)

# Example requirements coffee machine

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furthermore, whenever a coffee machine offers cappuccino, then it must offer coffee as well, while tea may only be offered by coffee machines for the European market (**requires** and **excludes** relations among features)

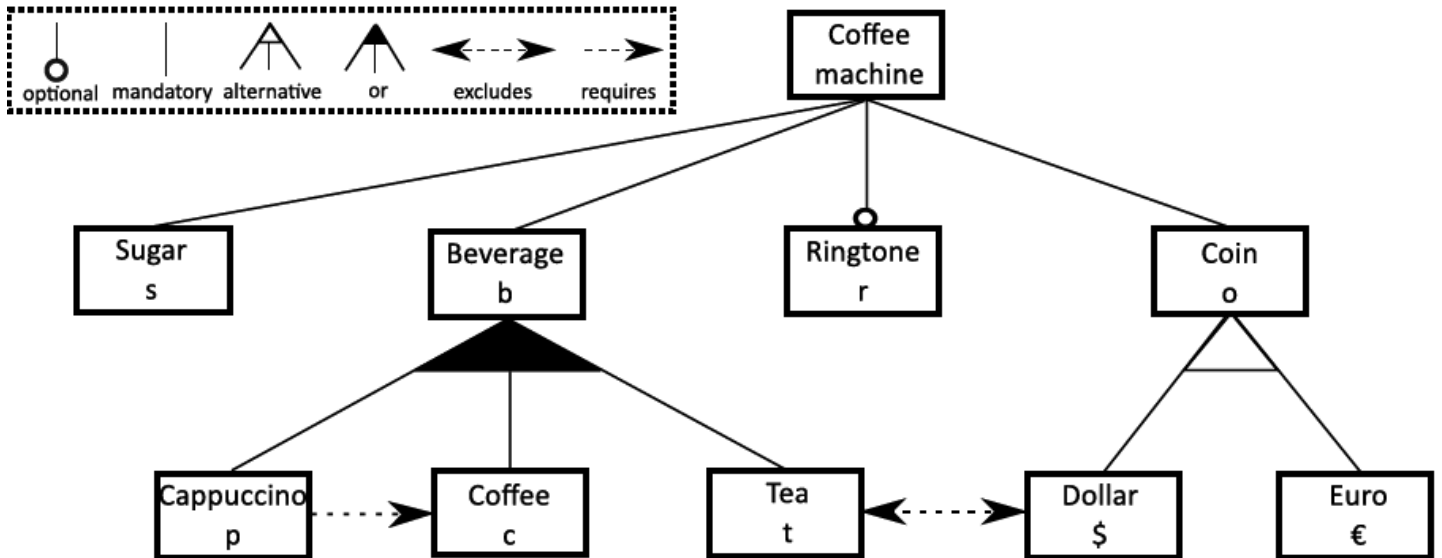
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4. Optionally, a ringtone may be rung after the coffee machine has delivered the chosen beverage (**optional** feature)

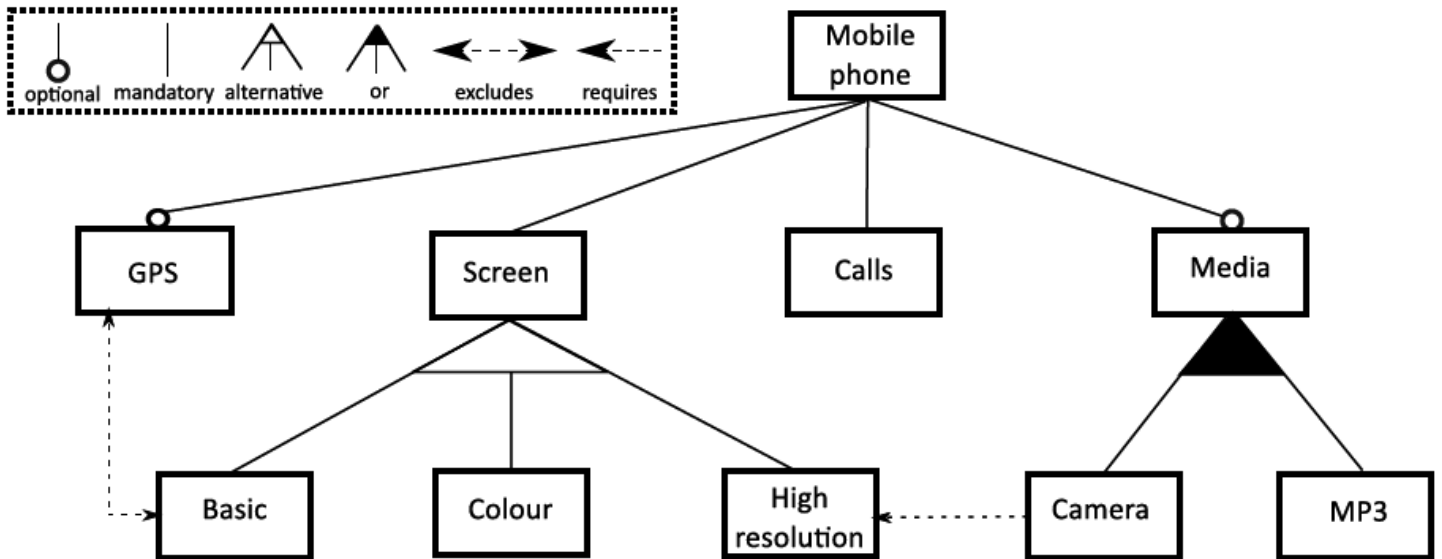
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4. Optionally, a ringtone may be rung after the coffee machine has delivered the chosen beverage (**optional** feature)
5. As soon as the user has taken her/his beverage, the coffee machine must return in its idle state

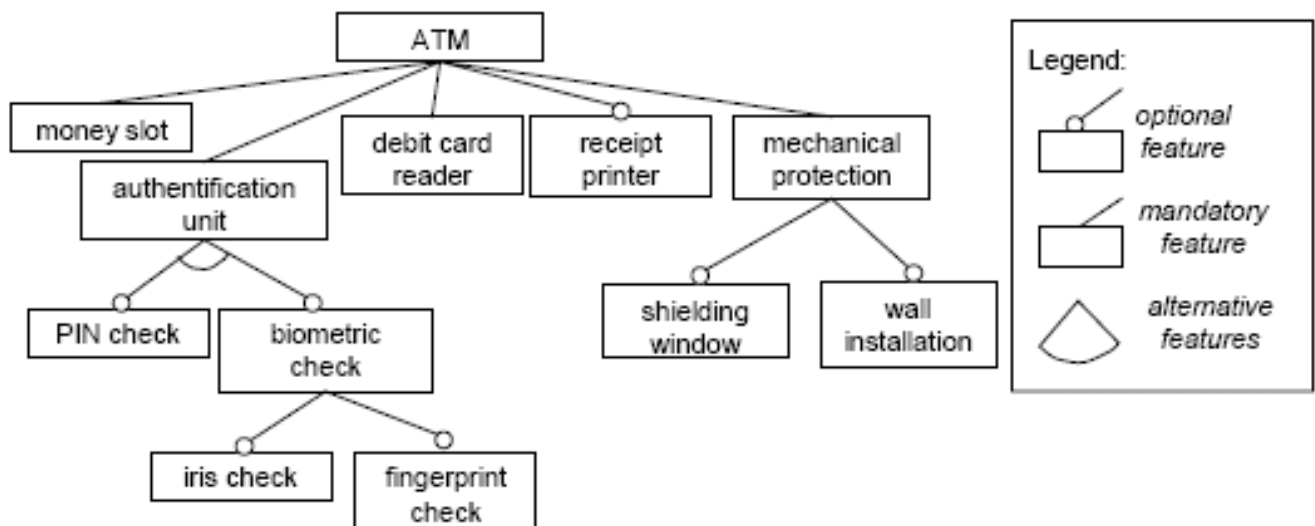
# Coffee machine feature diagram



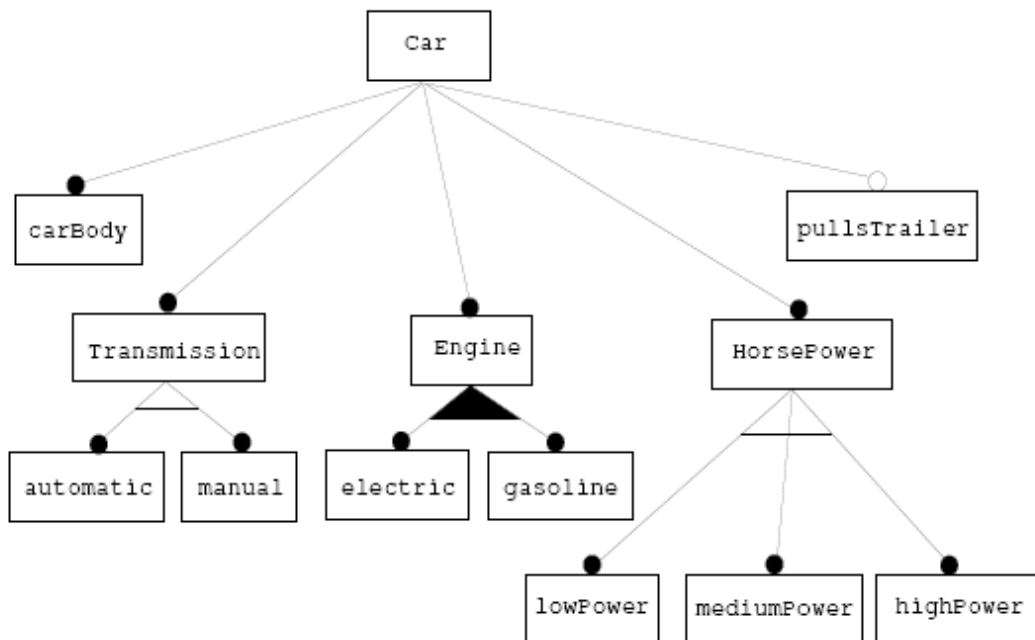
# Mobile phone feature diagram



# ATM feature diagram

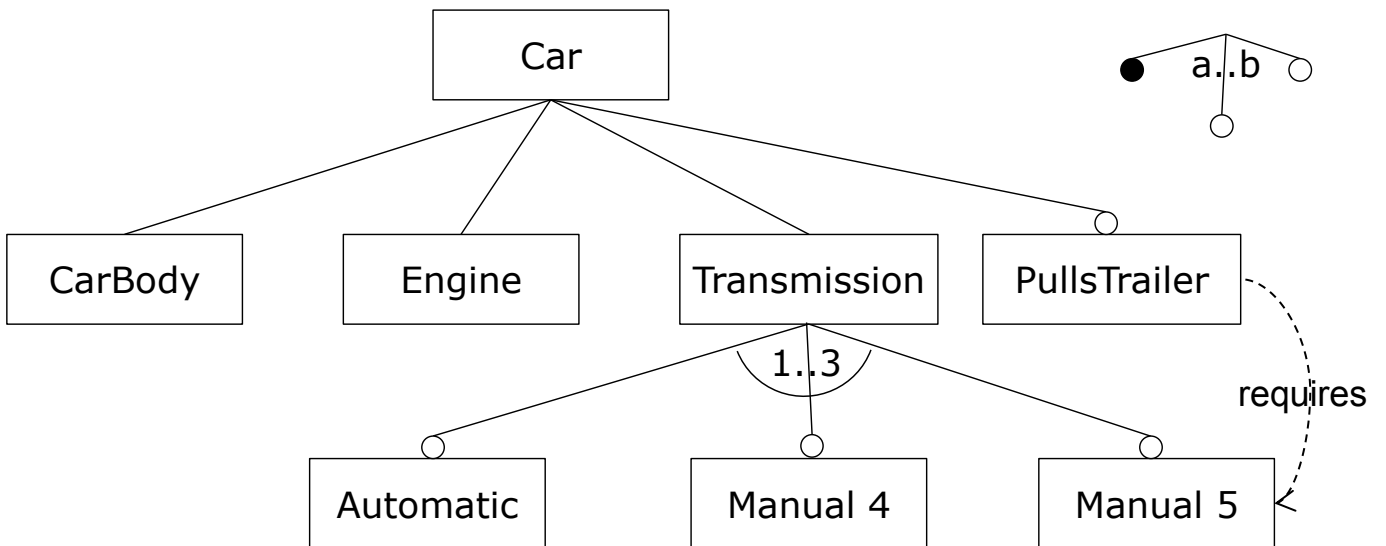
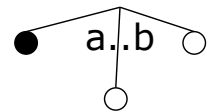


# Automotive feature diagram



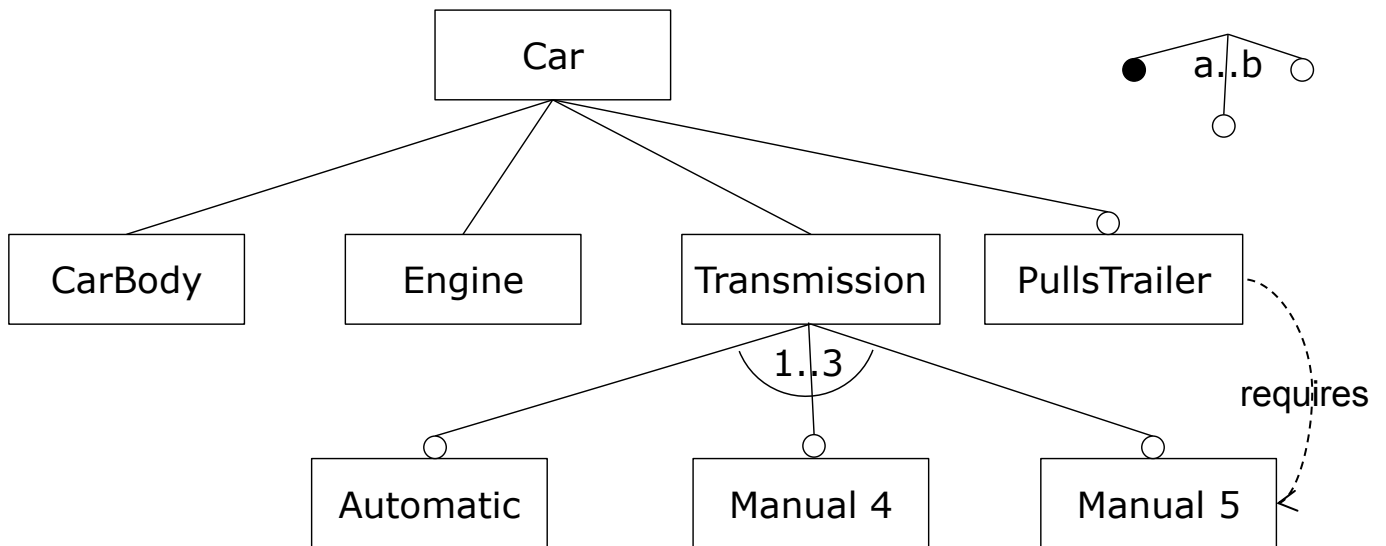
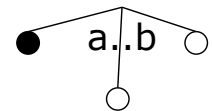
# How many products?

Pick a out of b features



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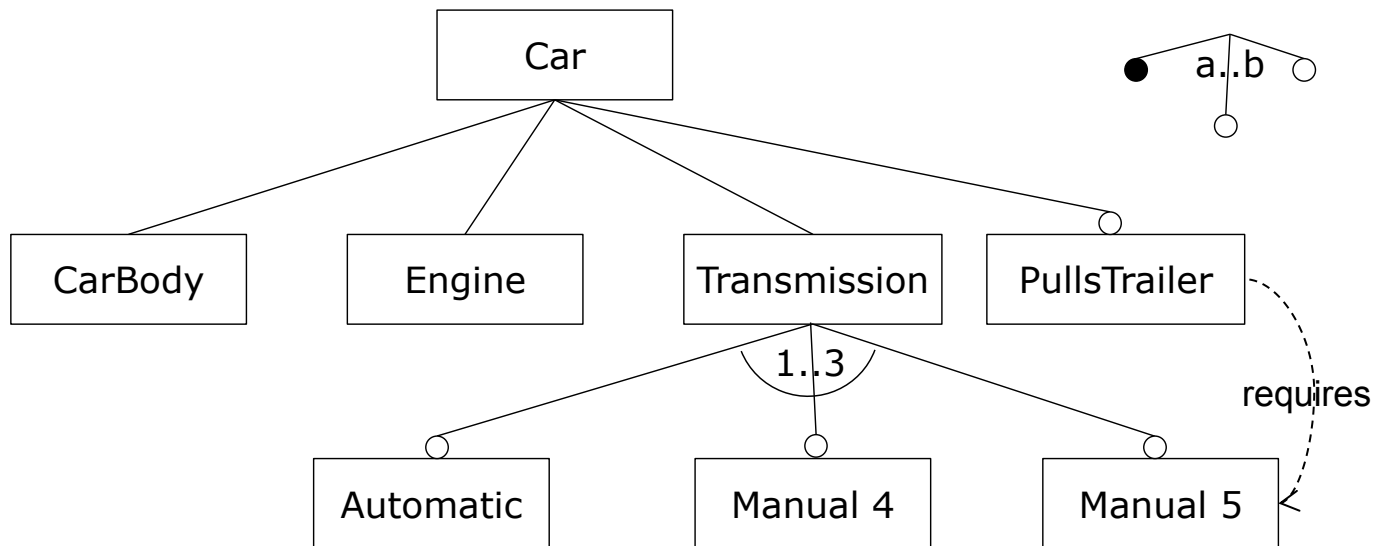
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Transmission contributes one of three options, multiplier = 3  
 Car contributes two options, multiplier = 2, but...

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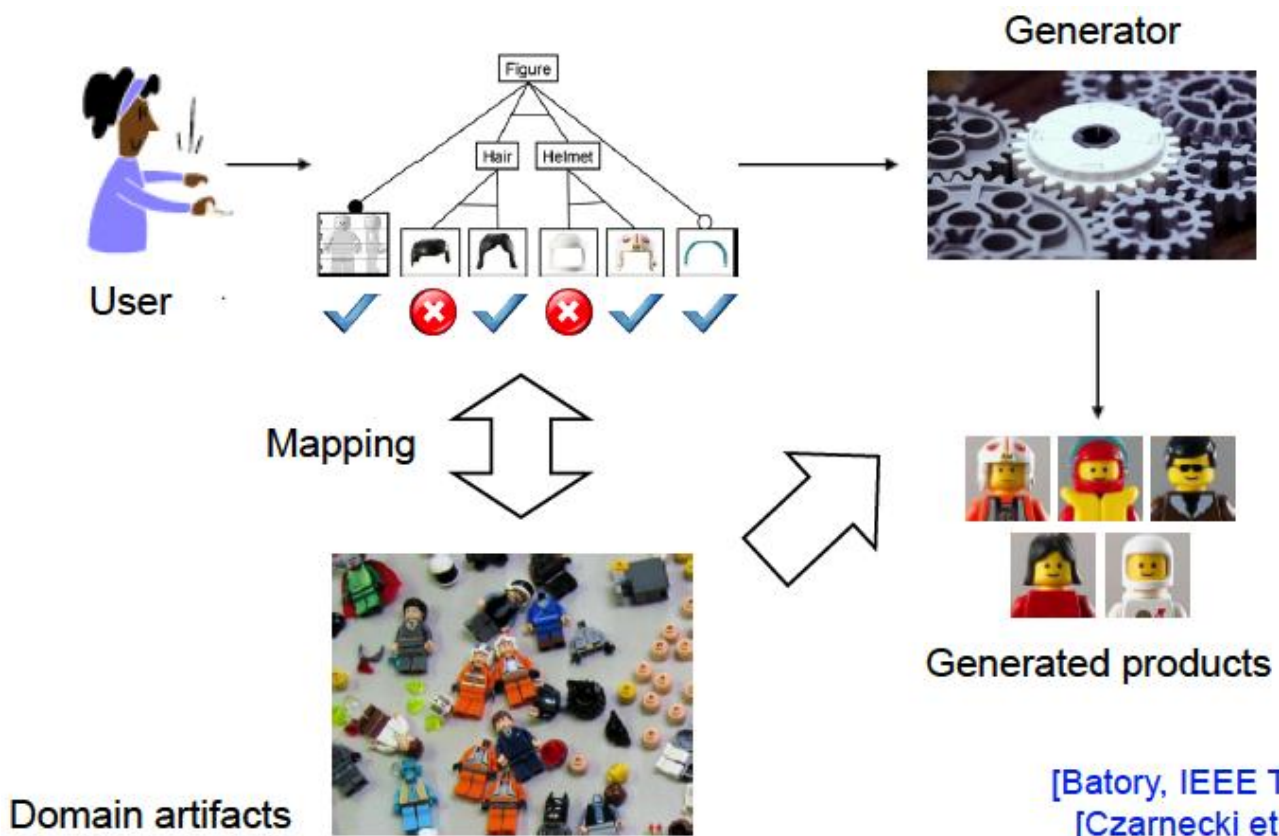


Transmission contributes one of three options, multiplier = 3  
 Car contributes two options, multiplier = 2, but...  
 ...actually only works in one case; thus: **4 different products**

# Applications of feature diagrams

- **Communicate** with stakeholders
- Identify objects for **reuse**
- Identify objects for **sales opportunities**
- Identify **cross-cutting concerns**
- Software **composition** and **deployment**
- etc.

# ... Automated product derivation

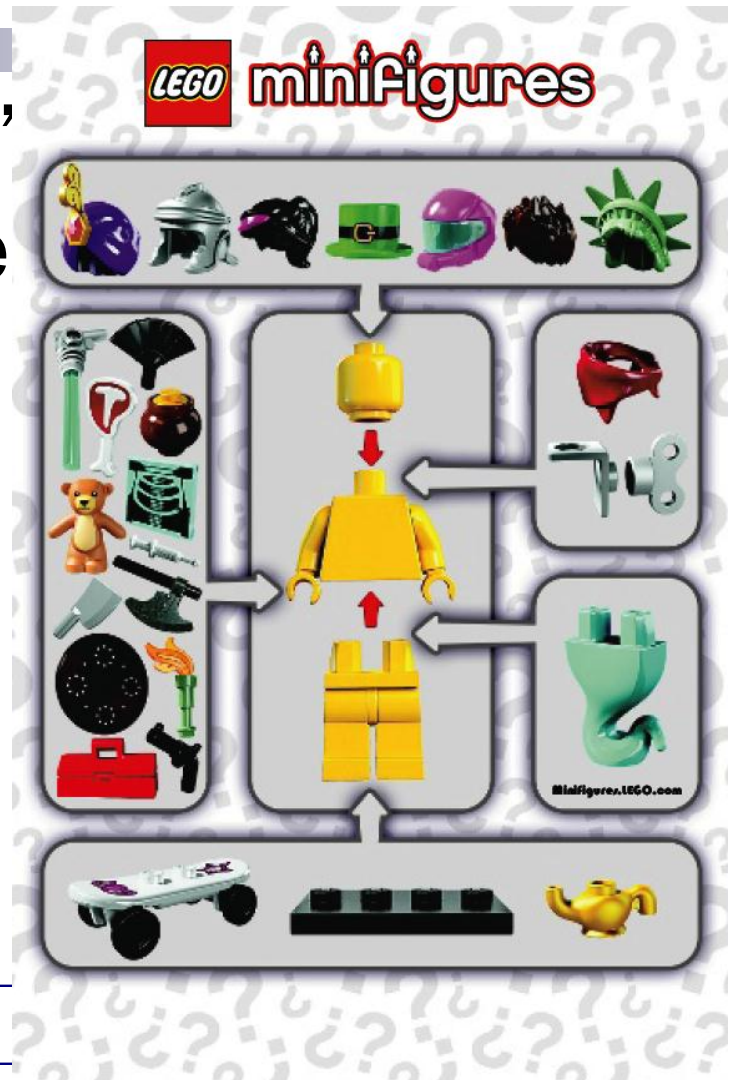


[Batory, IEEE TSE 2004]  
[Czarnecki et al., 2000]

# A product line of Lego minifigures (16 valid feature combinations)




‘Feature diagram’  
for configuring the  
16 valid products  
(allowing more...)




# VW features


Toon kleuren en bekleding voor **V8**


**Exterieur**




**Silver Leaf** Meerprijs € 0,-


Uni 

Metallic 


Metallic 


Pareffect 


**Interieur**




**Zonnebeige-Antraciet** Meerprijs € 3.250,-

Stof 

Leder 

Leder 

Leder 

Maak uw keuze uit het kleurenpalet door met de muis een kleur te selecteren.



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Your MacBook Air comes as standard with flash storage. Flash storage has no moving parts and provides faster responsiveness and enhanced durability.

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United Kingdom

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Make a date for a test drive with your BMW dealer.

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### Filter

> Reset filter

Budget

Vehicle type

- All
- Petrol
- Diesel
- Hybrid
- Electric Vehicle

Body type

- Saloon
- Touring
- Convertible
- Coupé
- Gran Turismo
- Sports Hatch
- Roadster
- Sports Activity Coupé
- Sports Activity Vehicle

Number of seats

30 Vehicles (465 Model variants)

1



**BMW 1 Series 3-door Sports Hatch (34)**  
from £ 17,775.00



**BMW 1 Series 5-door Sports Hatch (39)**  
from £ 18,305.00

2



**BMW 2 Series Coupé (14)**  
from £ 24,265.00

3



**BMW 3 Series Saloon (56)**  
from £ 23,550.00



**BMW 3 Series Touring (54)**  
from £ 24,865.00



**BMW 3 Series Gran Turismo (39)**  
from £ 29,200.00

# BMW product configurator



## 530d Sedan

Totaalprijs | Maandbedrag

**EUR 60.844,88**

- Configuratie- en prijsdetails bekijken
- Leasing & Financiering
- Configuratie opslaan
- Informatie over prijzen

Model **Carrosseriekleur, interieur en wielen** Verder ▶

### Carrosseriekleur kiezen

#### Uni



#### Metallic



### Interieur kiezen

#### Stof



#### Leder



Lichtmetalen wielen Sterspaak  
(122), 17"  
EUR 1.344,80

### Lichtmetalen wielen



Lichtmetalen wielen Sterspaak  
(122), 17"  
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# BMW feature packages



## 530d Sedan

Totaalprijs | Maandbedrag

**EUR 67.639,48**

- Configuratie- en prijsdetails bekijk
- Leasing & Financiering
- Configuratie opslaan
- Informatie over prijzen

Model Carrosseriekleur, interieur en wielen Verder >

Alpinweiss II  
EUR 0,00

Uni



Metallic



Stof/Leder Flashlight  
Anthrazit  
EUR 0,00

Stof



Leder

Leder

Lichtmetalen wielen M  
Dubbel spaak (172M), 19"  
EUR 1.556,62

Lichtmetalen wielen



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Dubbel spaak (172M), 19"  
EUR 1.556,62  
alleen met  
- M sportpakket (EUR 6.582,78)

# Smart constraints (1/2)

## smart roadster

roadster BRABUS

74 kW benzinemotor

74 kW benzinemotor


74 kW benzinemotor Xclusive

Exterieur  Interieur[6]  Vergroten


Kleuren    Standaard    Opties    Accessoires

Exterieur    Interieur

**bodypanels**



**tridion-veiligheidskooi**



## Prijs

Adviesprijs van de fabrikant	
Basisprijs:	31.450,00 €
tridion & bodypanels:	-,-- €
Interieur:	-,-- €
Opties:	-,-- €
Accessoires:	-,-- €
<b>Totaalprijs[1]:</b>	<b>31.450,00 €</b>

## Verder met...

Interieurkleur >

Brandstofverbruik[4] in l/100 km resp. km/l (gecombineerd): 5,2

# Smart constraints (2/2)

## smart roadster

roadster BRABUS

74 kW benzinemotor Xclusive



Exterieur  Interieur[6]  Vergroten

Kleuren

Standaard

Opties

Accessoires

Exterieur **Interieur**

### bodypanels



### tridion-veiligheidskooi



## Prijs

Adviesprijs van de fabrikant

Basisprijs:	35.670,00 €
tridion &	
bodypanels:	--- €
Interieur:	--- €
Opties:	--- €
Accessoires:	--- €

**Totaalprijs[1]: 35.670,00 €**

## Verder met...

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# Feature binding times

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  - etc.
- For lego:
  - Playing time 

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Allowed choices of variants at a specific variation point

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- **Mandatory variant**: if the variation point is selected, this variant *must* always be selected
- **Optional variant**: if the variation point is selected, this variant *may* be selected but it does not have to be
- **Alternative choice** i.e. a collection of at least two optional variants together with a [min...max] notation to indicate the permissible number of variants to be selected: if the variation point is selected, at least “min” variants *must* be selected while at most “max” variants *may* be selected

# Variability dependencies (2/2)

## Cross-tree constraints

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## Variability dependencies (2/2)

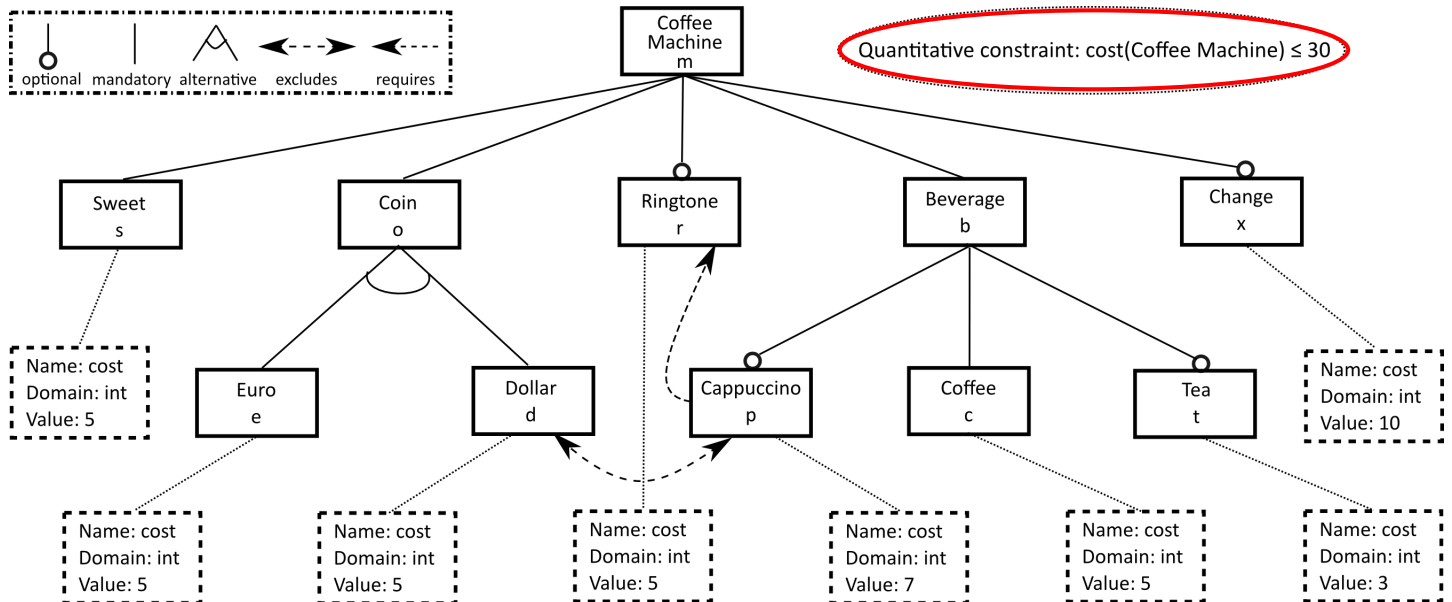
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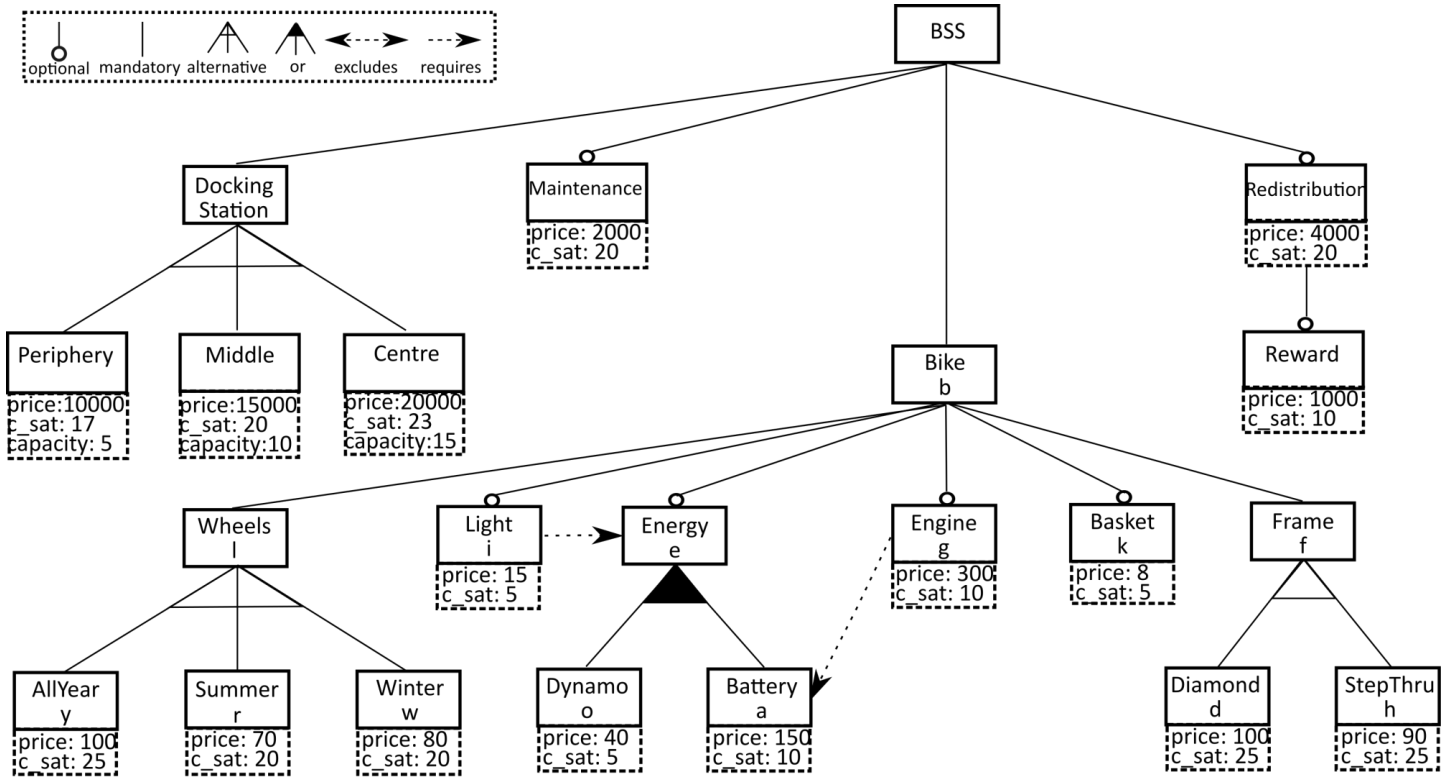
### Quantitative constraints

- **Feature attributes (non-functional)**:  $cost(\text{feature}) = 7$ , etc.
- $cost(\text{product}) = \sum\{cost(\text{feature}) \mid \text{feature} \in \text{product}\}$

# Attributed feature model (1/2)



# Attributed feature model (2/2)



# Domain Requirement Engineering

- During domain RE, the **requirements for the entire SPL** are defined

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- During domain RE, the **requirements for the entire SPL** are defined
- Basis for: (i) developing the entire SPL family  
(ii) defining the requirements of each product

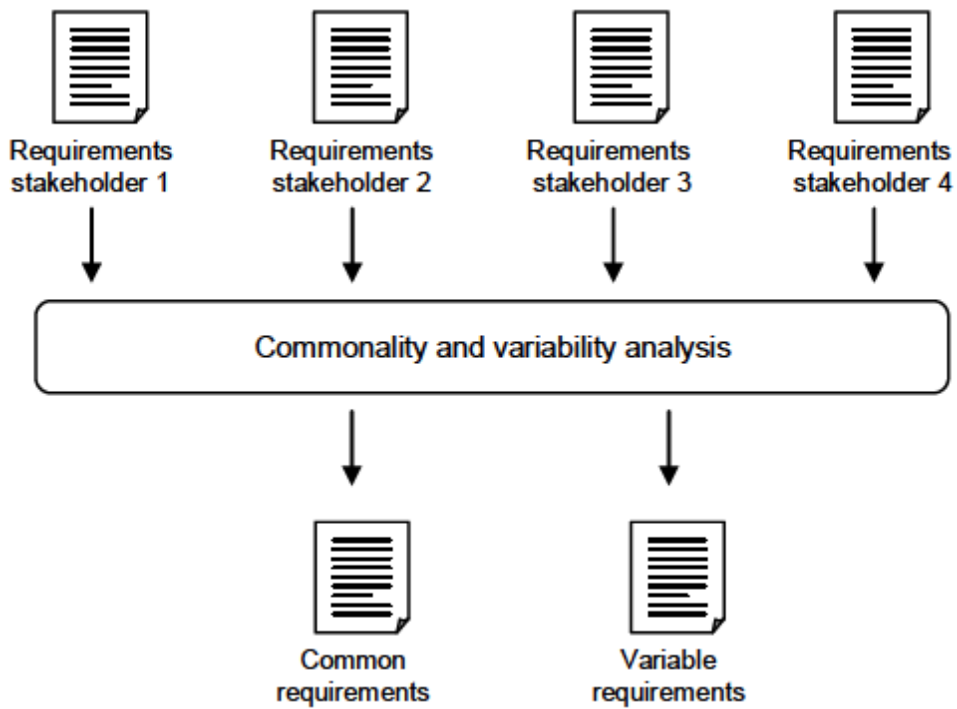
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these activities have the **same goals plus**, in addition, the **goal to define the SPL variability**

# Elicitation of requirements variability



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# So why have variability?

## ■ Flexibility to deliver

- Different versions of products based on the **same trunk** of code (Windows XP professional, Windows XP Home edition, etc.)
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“We always have 126,000,000 different bicycles in store!

(but only the parts for 1,000...)”

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  2. Carolina is accepted into the 3-way call

# Weakness: complexity/scalability

with **33** optional, independent features



a unique product for every

---

## person on this planet

---

Examples by C. Kästner (CMU, Pittsburgh, USA) <sup>44</sup>

320 optional, independent  
features

more possible products than estimated

atoms in the universe



# (Behavioural) variability analysis

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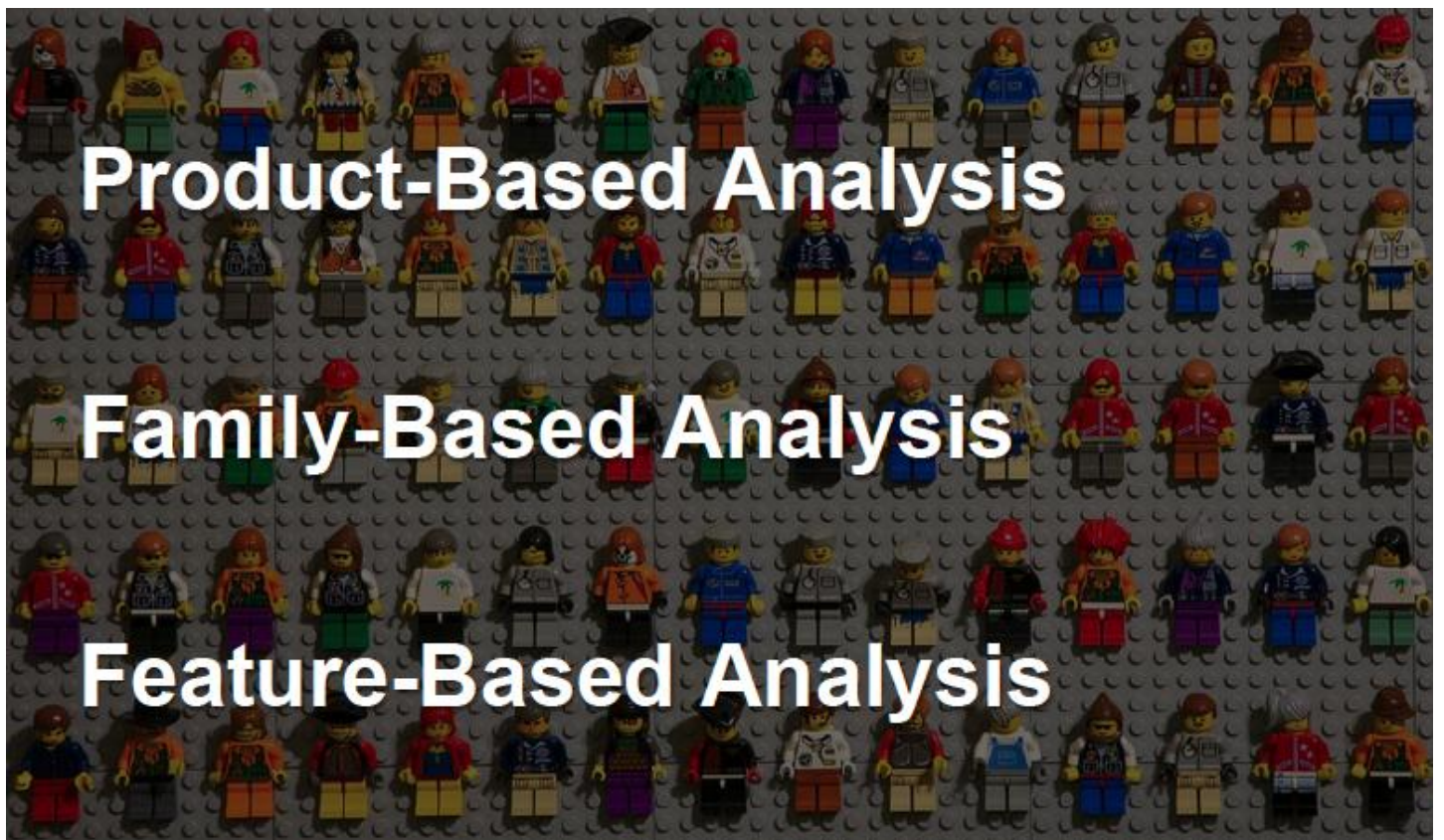
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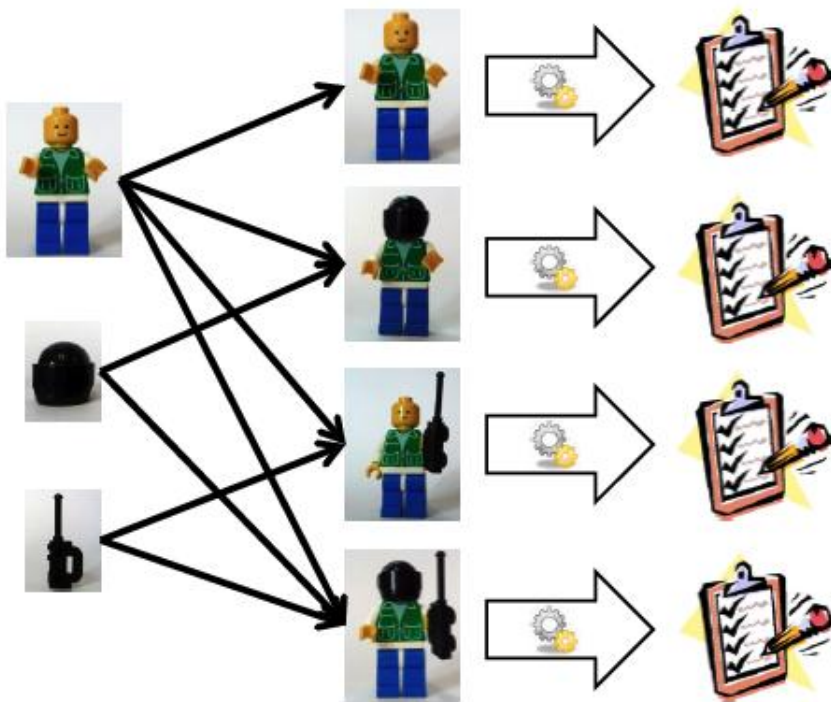
- Mainstream formal methods do *not* consider **variability** directly
- Formal methods that *have* been applied in SPLE mainly focus on structural rather than **behavioural** properties (feature model analysis: e.g. dead features, false optional features, etc.)

# Variability analysis strategies

(type checking, static analysis, model checking, theorem proving, testing, etc.)

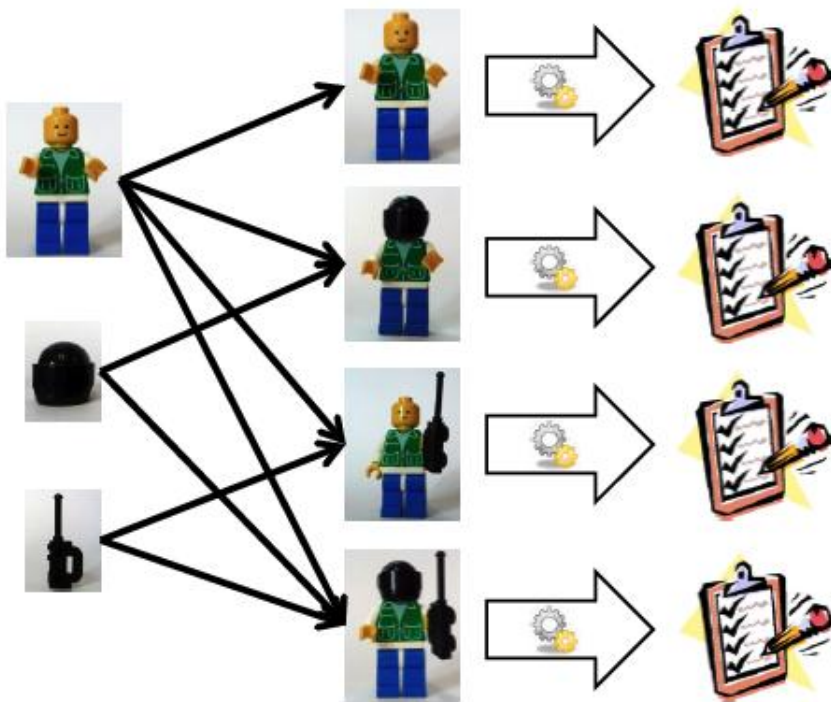


# Product-based analysis



$O(2^n)$  for  $n$  features

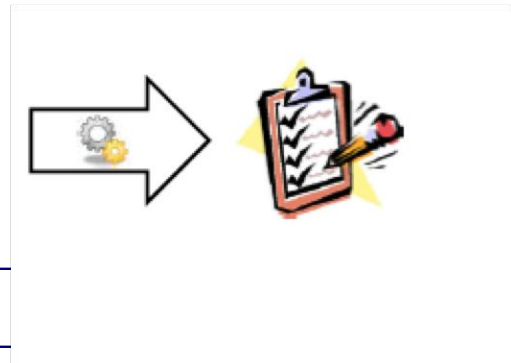
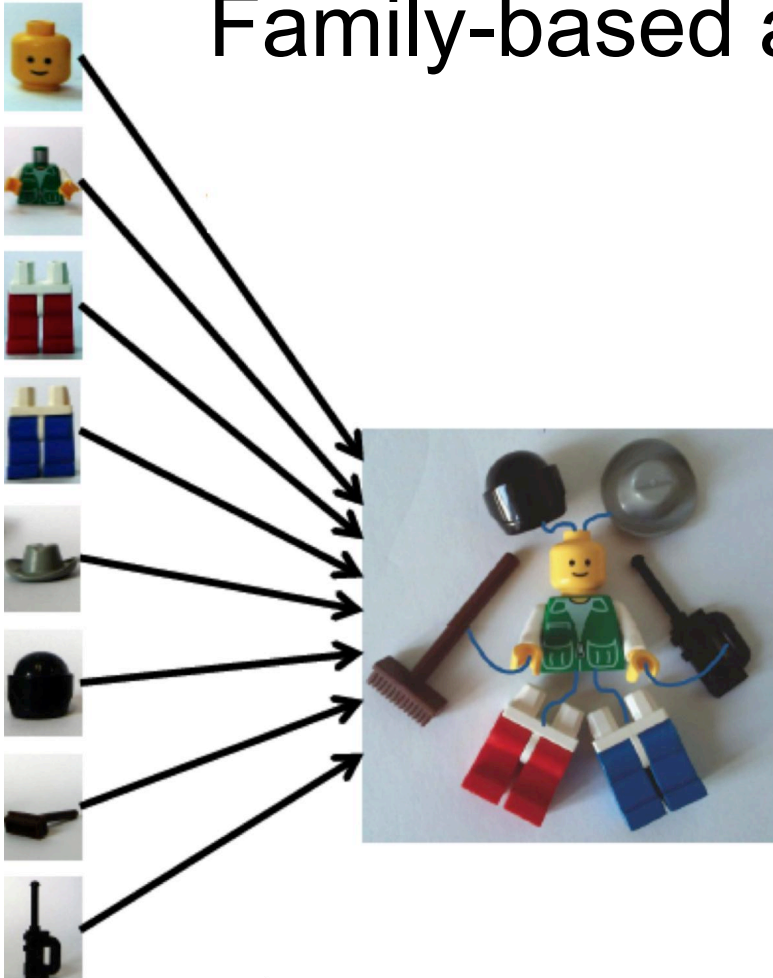
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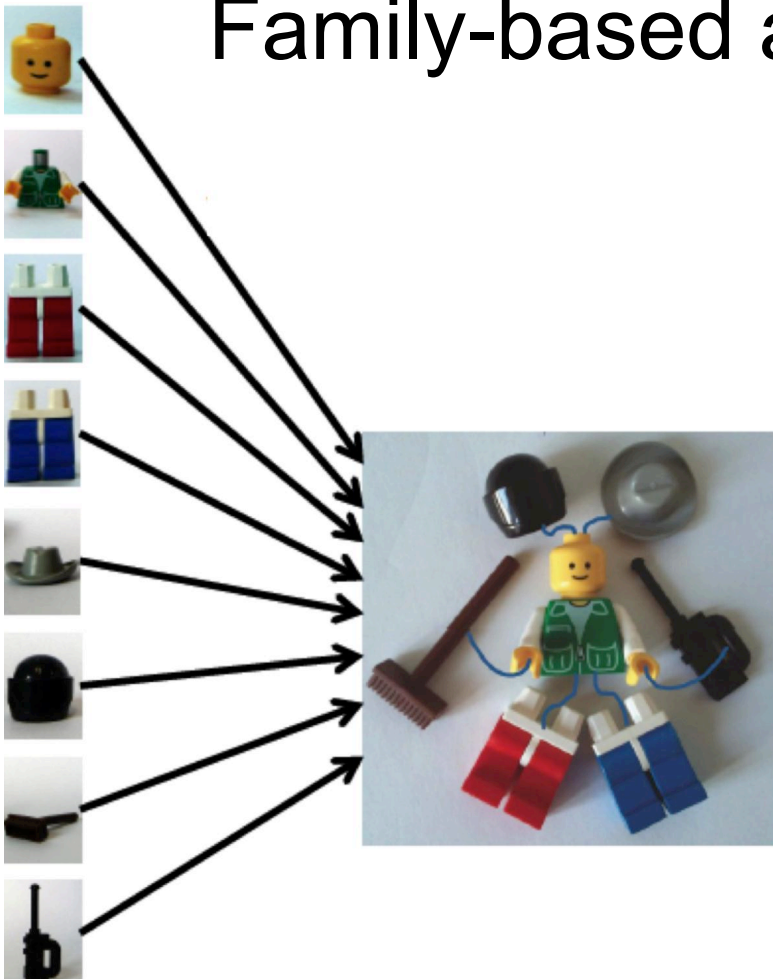
Simple approach  
Standard tools available  
Infeasible for large product sets  
Same behaviour / code verified numerous times

$O(2^n)$  for  $n$  features

# Family-based analysis



# Family-based analysis



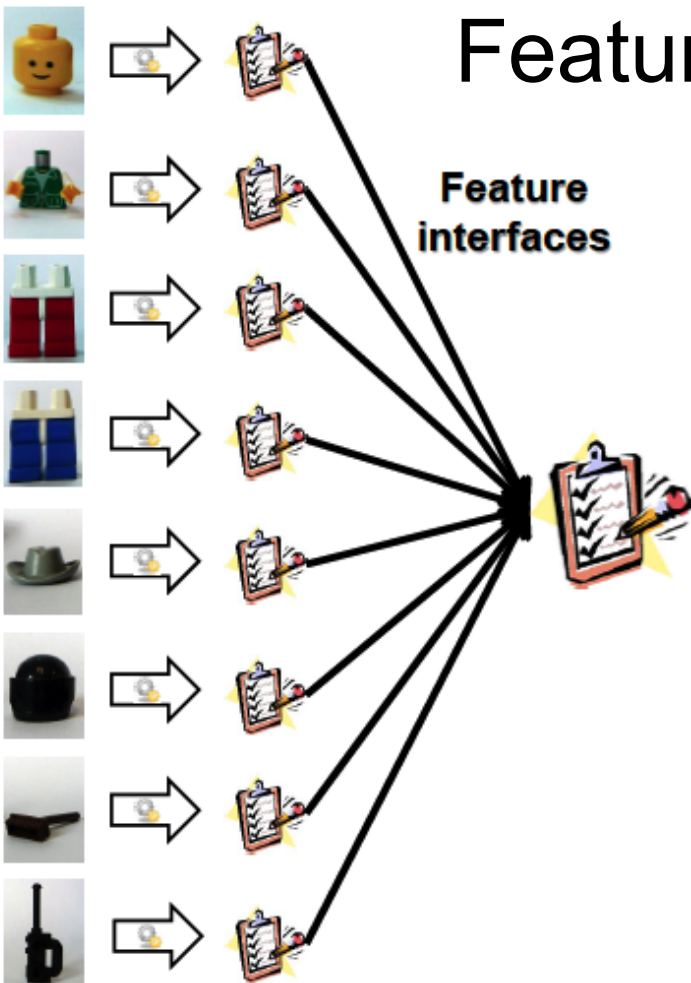
Beneficial for many products with substantial similarities

Generates complex analysis tasks

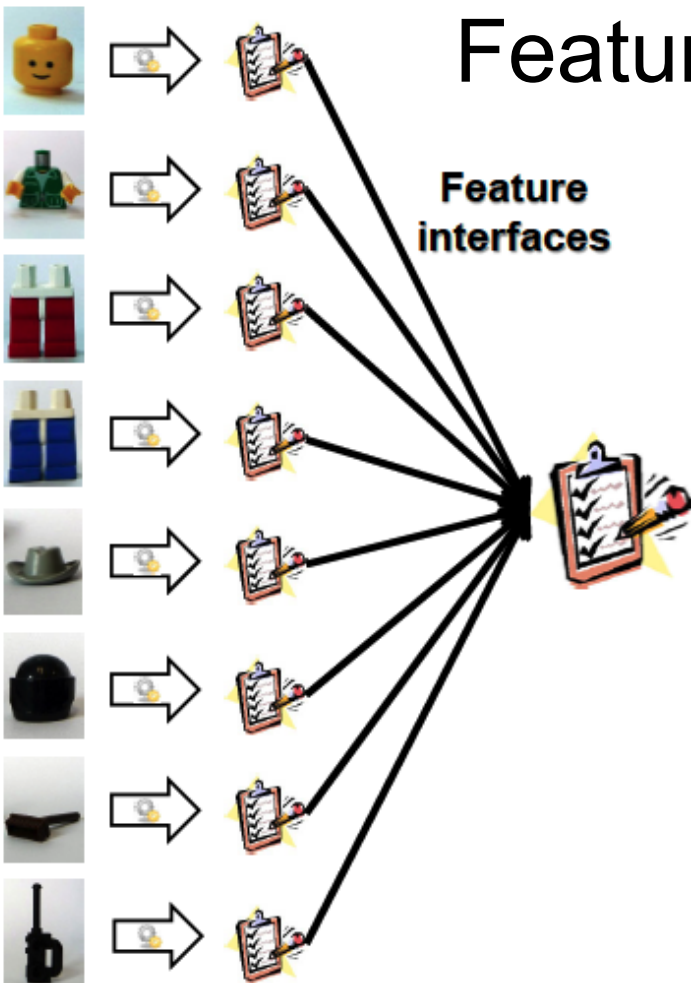
**Requires (compact) family metamodels**



# Feature-based analysis



# Feature-based analysis



## Modelling and Analysing Variability

Lift success stories known for single systems (products) to sets of products (families) by exploiting **variability** modelling and analysis

⇒ challenges known models and tools by potentially high number of different products, each giving rise to a large state space in general

- Featured Transition Systems (FTS)

*Classen et al. @ ICSE'10, IEEE TSE, 2013, Sci. Comput. Program., 2014*

SNIP/fPROMELA/fLTL, fNuSMV/fSMV/fCTL, ProVeLines

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Variability Model Checker VMC/v-ACTL

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- UML SPL profile, PL-CCS, variable I/O automata, feature nets, etc.

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## FTS for SPLE

Recall (atomic propositions used for verification purposes):

De Nicola & Vaandrager © *J. ACM*, 1995

A **Doubly-Labelled Transition System** ( $L^2TS$ ) is a sextuple  $(Q, A, \bar{q}, \rightarrow, AP, L)$  with states  $Q$ , actions  $A$ , initial state  $\bar{q}$ , transitions  $\rightarrow \subseteq Q \times A \times Q$ , atomic propositions  $AP$ , and labeling function  $L : S \rightarrow 2^{AP}$

An FTS adds to this a feature model and feature expressions:

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A **Featured Transition System** (FTS) is an octuple  $(Q, A, \bar{q}, \rightarrow, AP, L, FD, \gamma)$  with *underlying*  $L^2TS$   $(Q, A, \bar{q}, \rightarrow, AP, L)$ , feature diagram  $FD$  over a set  $\mathbb{F}$  of features, and total function  $\gamma : \rightarrow \rightarrow \mathbb{B}(\mathbb{F})$  labelling each transition with a **feature expression**, i.e. a Boolean expression over the features

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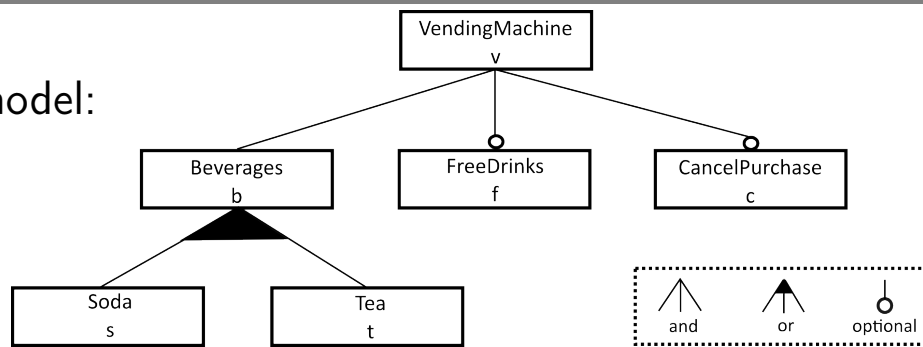
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Feature model:

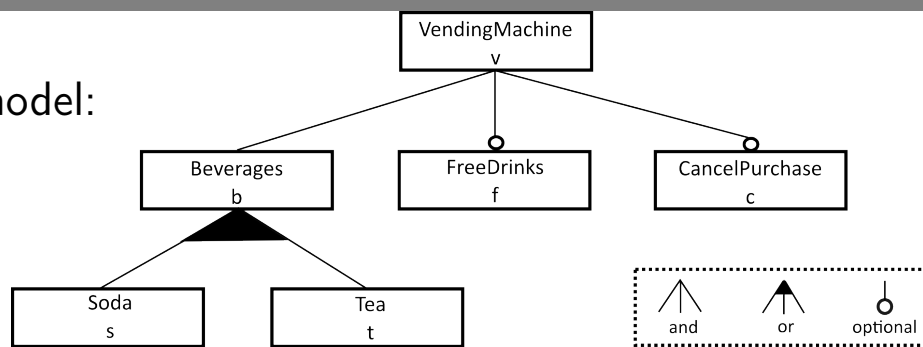


12 valid products

e.g.  $\{v, b, s, t\}$ ,  $\{v, b, s, c\}$

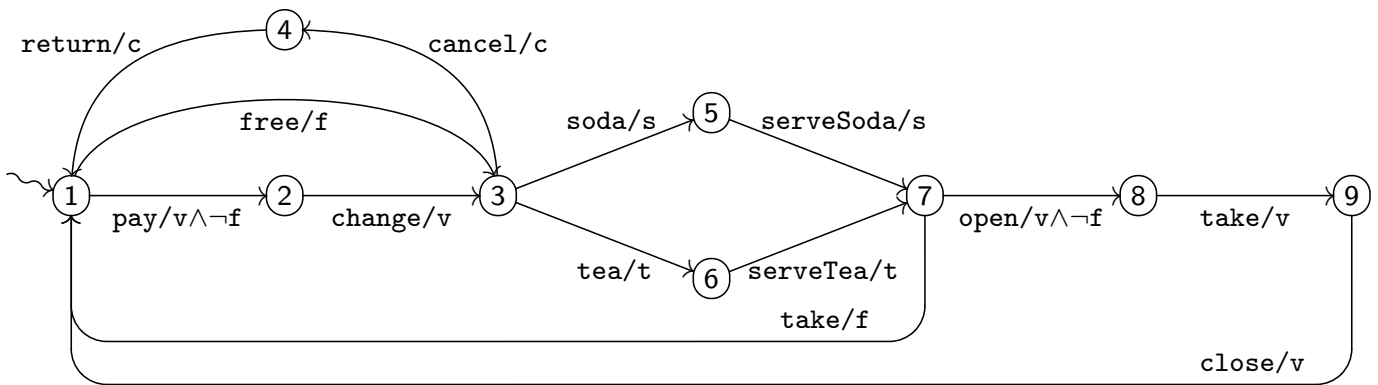
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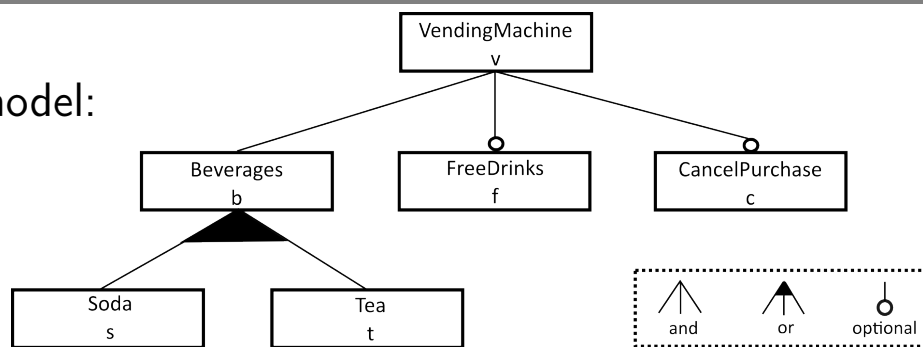
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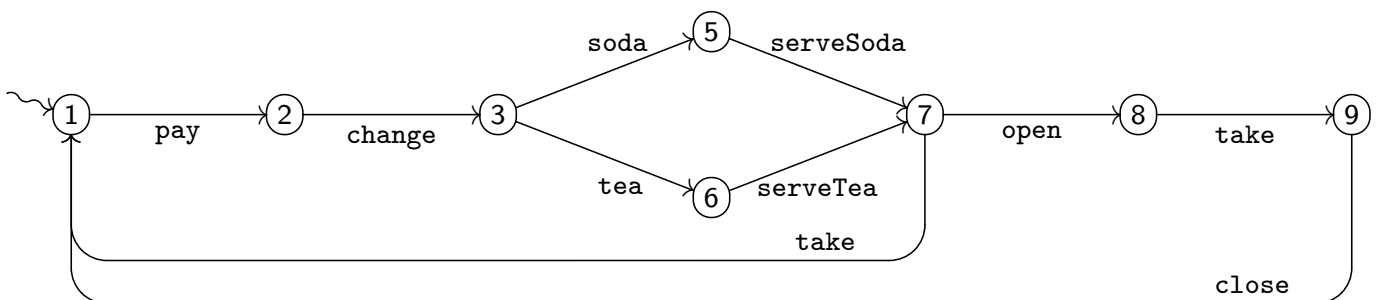


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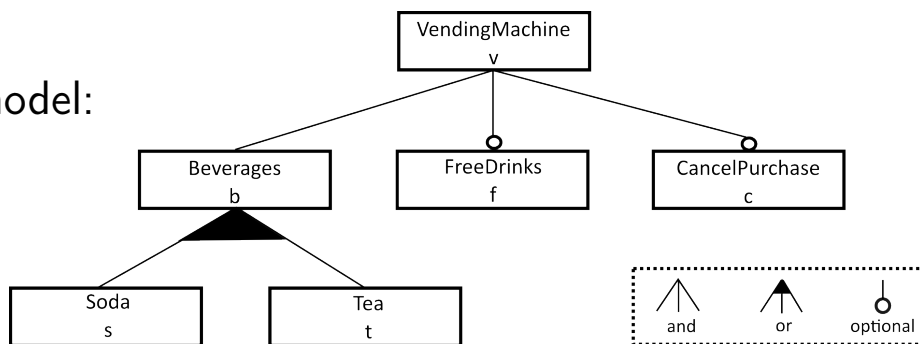


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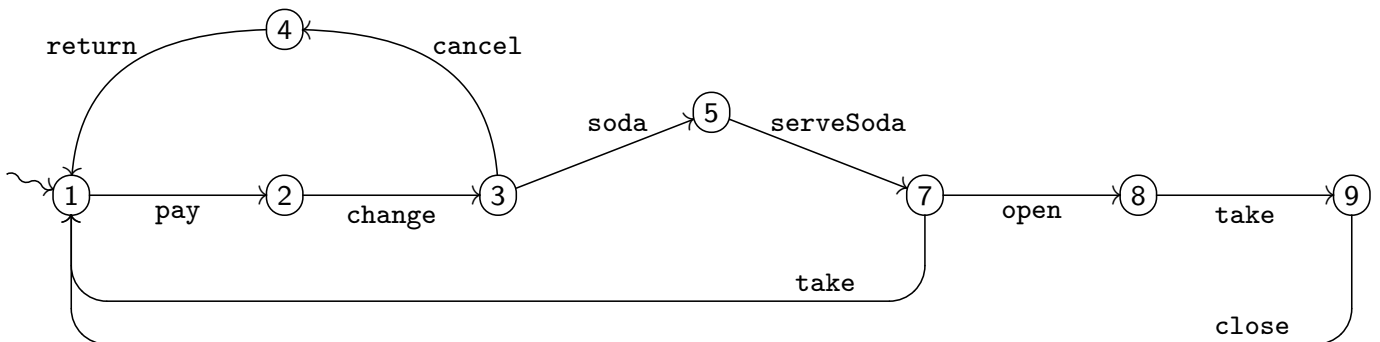


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## Dedicated FTS model checker SNIP (now ProVeLines)

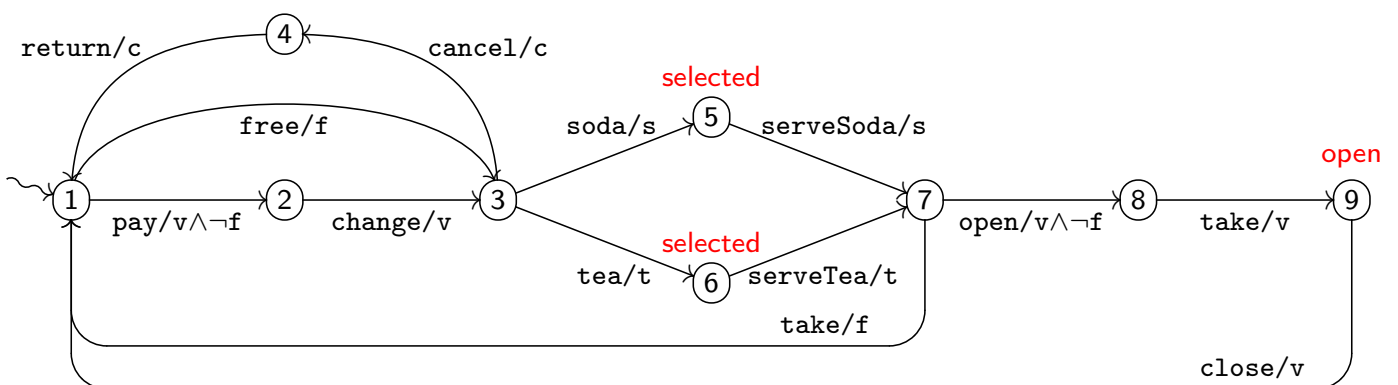
fPROMELA:

```

typedef features {
  bool v;
  bool f;
  ...
};
features F;
    
```

```

gd :: F.v && !F.f;
      pay;
      :: F.f;
      free;
      :: else;
      skip;
dg;
    
```



fLTL:

$[\neg f] \square (\text{selected} \Rightarrow \diamond \text{open})$

Similarly fNuSMV with fSMV/fCTL

## MTS<sub>v</sub> for SPLE

Main ingredient: Modal Transition Systems (MTS)

- LTS distinguishing possible (**may**) and required (**must**) transitions

Larsen & Thomsen @ LICS'88

- Recognised as a useful model to describe in a **compact** way the possible **behaviour** of all the products (LTS) of a product family

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👉 MTS cannot model variability constraints regarding **alternative** features, nor regarding **requires/excludes** inter-feature relations, resulting in several variants and extensions

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👍 Our solution: add a set of **variability constraints** to the MTS to be able to decide which derivable products (LTS) are valid ones

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## MTS with...

Recall:

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Next we define MTS with variability constraints:

A **Modal Transition System** (MTS) is a quintuple  $(Q, A, \bar{q}, \rightarrow_{\square}, \rightarrow_{\diamond})$  such that  $(Q, A, \bar{q}, \rightarrow_{\square} \cup \rightarrow_{\diamond})$  is an LTS, called its underlying LTS

An MTS has two distinct transition relations

1. **may** transition relation  $\rightarrow_{\diamond} \subseteq Q \times A \times Q$ : **possible** transitions
2. **must** transition relation  $\rightarrow_{\square} \subseteq Q \times A \times Q$ : **required** transitions

By definition, any required transition is also possible:  $\rightarrow_{\square} \subseteq \rightarrow_{\diamond}$   
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## ... variability constraints (accepted by VMC)

Variability constraints of form **ALT**ernative, **EXC**ludes, **REQ**uires, etc.

$a_1$  **ALT**  $\dots$  **ALT**  $a_n$  : precisely one among the  $n \geq 2$  actions  $a_1, \dots, a_n$  is reachable in  $\mathcal{L}$  (i.e. is the label of a reachable transition)

$b_1$  **OR**  $\dots$  **OR**  $b_n$ , where  $b_i$  is either  $a_i$  or  $\neg a_i$  : at least one among the conditions on  $n \geq 2$  actions  $b_1, \dots, b_n$  holds, i.e.  $b_i = a_i$  is reachable in  $\mathcal{L}$  or  $b_i = \neg a_i$  is not reachable in  $\mathcal{L}$

$a_1$  **EXC**  $a_2$  : at most one of the actions  $a_1$  and  $a_2$  is reachable in  $\mathcal{L}$

$a_1$  **REQ**  $a_2$  : action  $a_2$  is reachable in  $\mathcal{L}$  whenever  $a_1$  is reachable in  $\mathcal{L}$

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## Derive products (implemented in VMC)

A **product LTS** is obtained from a family MTS in the following way:

1. include **all** (reachable) must transitions and
  2. include **subset** of the (reachable) optional transitions, remove rest
  3. satisfy assumptions of **coherence** and **consistency**
  4. satisfy **variability constraints**
- ⇒ Each selection gives rise to a different variant

Let  $(Q, A, \bar{q}, \delta^\diamond, \delta^\square, \Upsilon)$  be a coherent MTS<sub>v</sub>, i.e.  $\exists \xrightarrow{a} \implies \nexists \xrightarrow{a}$

The set  $\{\mathcal{P}_i = (Q_i, A, \bar{q}, \delta_i) \mid i > 0\}$  of **product LTS** is obtained by considering each pair of  $Q_i \subseteq Q$  and  $\delta_i \subseteq \delta^\diamond \cup \delta^\square$  to be defined s.t.

1. every  $q \in Q_i$  is reachable in  $\mathcal{P}_i$  from  $\bar{q}$  via transitions from  $\delta_i$
2. there exists no  $(q, a, q') \in \delta^\square \setminus \delta_i$  such that  $q \in Q_i$
3. LTS is consistent: both  $\xrightarrow{a} \rightsquigarrow \xrightarrow{a}$  and  $\xrightarrow{a} \not\rightsquigarrow \xrightarrow{a}$  not allowed
4.  $\mathcal{P}_i$  satisfies all variability constraints in  $\Upsilon$

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- ⇒ Each selection gives rise to a different variant

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The set  $\{\mathcal{P}_i = (Q_i, A, \bar{q}, \delta_i) \mid i > 0\}$  of **product LTS** is obtained by considering each pair of  $Q_i \subseteq Q$  and  $\delta_i \subseteq \delta^\diamond \cup \delta^\square$  to be defined s.t.

1. every  $q \in Q_i$  is reachable in  $\mathcal{P}_i$  from  $\bar{q}$  via transitions from  $\delta_i$
2. there exists no  $(q, a, q') \in \delta^\square \setminus \delta_i$  such that  $q \in Q_i$
3. LTS is consistent: both  $\xrightarrow{-a} \rightsquigarrow \xrightarrow{a}$  and  ~~$\xrightarrow{a}$~~  not allowed
4.  $\mathcal{P}_i$  satisfies all variability constraints in  $\Upsilon$

## MTS<sub>v</sub> equally expressive as FTS

ter Beek et al. © SEFM'15, *Sci. Comput. Program.*, 2019

Theorem (FTS2MTS<sub>v</sub> transformation is sound and complete)

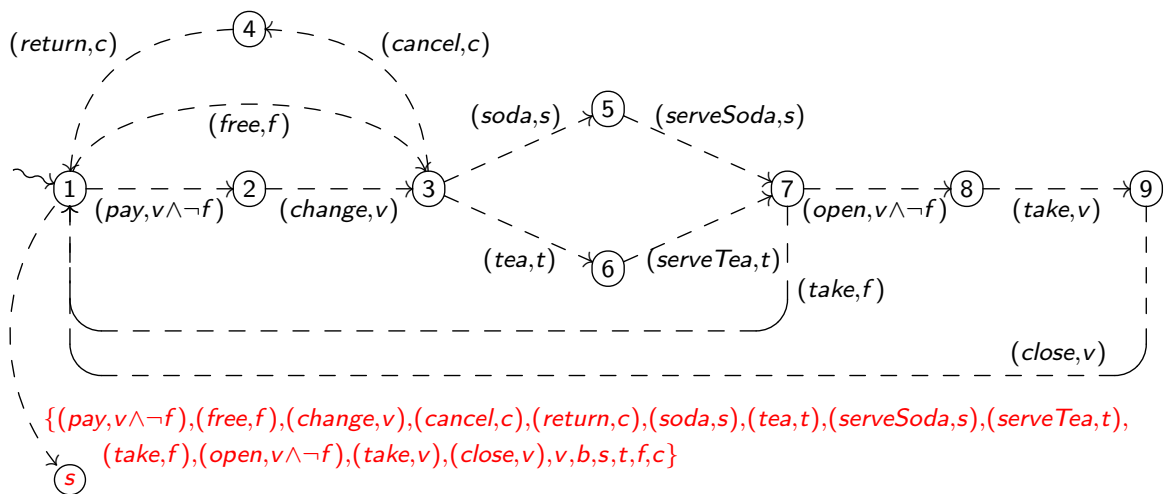
*Let  $\mathcal{F}$  be an FTS and let  $\mathcal{M}$  be the MTS<sub>v</sub> generated from  $\mathcal{F}$  according to the FTS2MTS<sub>v</sub> model transformation algorithm. Then the sets of derived variants  $\text{lts}(\mathcal{F})$  and  $\text{lts}(\mathcal{M})$  coincide, up to dummy transitions and action relabelling.*

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$$\Gamma = \{(\text{pay}, v \wedge \neg f) \Leftrightarrow (v \wedge \neg f), (\text{free}, f) \Leftrightarrow f, (\text{change}, v) \Leftrightarrow v, (\text{cancel}, c) \Leftrightarrow c, (\text{return}, c) \Leftrightarrow c, (\text{soda}, s) \Leftrightarrow s, (\text{tea}, t) \Leftrightarrow t, (\text{serveSoda}, s) \Leftrightarrow s, (\text{serveTea}, t) \Leftrightarrow t, (\text{take}, f) \Leftrightarrow f, (\text{open}, v \wedge \neg f) \Leftrightarrow (v \wedge \neg f), (\text{take}, v) \Leftrightarrow v, (\text{close}, v) \Leftrightarrow v\}$$

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## Dedicated MTS<sub>v</sub> Variability Model Checker VMC

VMC builds on optimization of UMC (input: UML state machines)

ter Beek *et al.* © *Sci. Comput. Program.*, 2011

VMC: bounded, on-the-fly model checking, providing **explanations**

↙ Biere *et al.* © TACAS'99

VMC accepts as input a specification in (value-passing) modal process algebra, possibly with additional variability constraints

- interactively explore the model (MTS)
- derive and explore (all) the model's valid products (LTS)
- visualize the model/products graphically as MTS/LTS
- verify v-CTL properties over MTS/LTS
- interactively explain why a property is (not) satisfied

Model checking of v-CTL formulae on MTS can be achieved in a complexity that is **linear** w.r.t. the state space size

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## Variability analysis strategies supported by VMC

VMC offers both product-based and family-based variability analyses:

1. The actual set of all valid product behaviour can explicitly be generated and the resulting LTS can all be verified against one and the same logic property (expressed in Action-based CTL)
2. A logic property (expressed in **variability-aware** ACTL) can directly be verified against the MTS, relying on the fact that under certain syntactic conditions validity over the MTS implies validity of the same property for all its derived products (more later)

De Nicola, Vaandrager © J. ACM, 1995

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VMC v6.4 is freely usable online:

<http://fmt.isti.cnr.it/vmc/>

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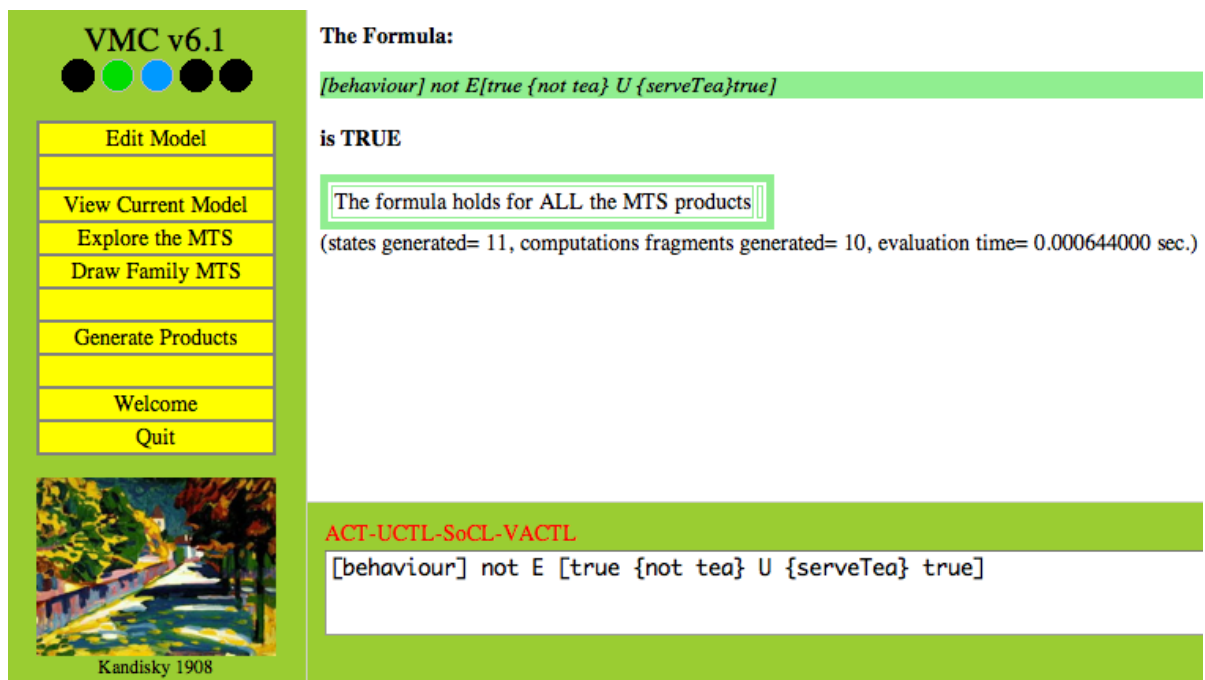
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## Vending Machine: family-based analysis

VMC notifies whenever preservation of an analysis result is applicable




The screenshot shows the VMC v6.1 interface. On the left is a menu with options: Edit Model, View Current Model, Explore the MTS, Draw Family MTS, Generate Products, Welcome, and Quit. Below the menu is a painting by Kandinsky from 1908. On the right, the 'The Formula:' section displays the formula  $[behaviour] \text{ not } E[true \{not \text{ tea}\} U \{serveTea\}true]$  and states it is TRUE. A highlighted box contains the text 'The formula holds for ALL the MTS products'. Below this, it shows performance metrics: (states generated= 11, computations fragments generated= 10, evaluation time= 0.000644000 sec.). At the bottom, the 'ACT-UCTL-SoCL-VACTL' section shows the formula  $[behaviour] \text{ not } E [true \{not \text{ tea}\} U \{serveTea\} true]$ .

*It is not possible that serveTea occurs without being preceded by tea*

## Vending Machine: product-based analysis

VMC lists for each product the action labels of all may transitions that have been preserved (as must transitions) in that product



**Evaluation of the formula "[behaviour] [pay] AF {takePaid} true" on all family products**

<a href="#">product_1+Soda+open+pay+soda</a>	Formula evaluates	TRUE
<a href="#">product_10+CancelPurchase+FreeDrinks+Tea+cancel+free+takeFree+tea</a>	Formula evaluates	TRUE
<a href="#">product_11+FreeDrinks+Soda+Tea+free+soda+takeFree+tea</a>	Formula evaluates	TRUE
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<a href="#">product_3+Tea+open+pay+tea</a>	Formula evaluates	TRUE
<a href="#">product_4+CancelPurchase+Tea+cancel+open+pay+tea</a>	Formula evaluates	FALSE
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<a href="#">product_7+FreeDrinks+Soda+free+soda+takeFree</a>	Formula evaluates	TRUE
<a href="#">product_8+CancelPurchase+FreeDrinks+Soda+cancel+free+soda+takeFree</a>	Formula evaluates	TRUE
<a href="#">product_9+FreeDrinks+Tea+free+takeFree+tea</a>	Formula evaluates	TRUE

**Logic Formula for all Products**

[behaviour] [pay] AF {takePaid} true

*Whenever pay occurs, eventually takePaid occurs*

## Recall: Action-based CTL

**Action formulae** (Boolean compositions of actions, denoted by  $\chi$ ),  
**state formulae** ( $\phi$ ) and **path formulae** ( $\pi$ )

$$\begin{aligned}\phi &::= \text{true} \mid \neg\phi \mid \phi \wedge \phi \mid [\chi]\phi \mid \langle\chi\rangle\phi \mid E\pi \mid A\pi \mid \\ &\quad \mu Y.\phi(Y) \mid \nu Y.\phi(Y) \\ \pi &::= X\{\chi\}\phi \mid [\phi\{\chi\}U\{\chi'\}\phi'] \mid [\phi\{\chi\}W\{\chi'\}\phi'] \mid \\ &\quad [\phi\{\chi\}U\phi'] \mid [\phi\{\chi\}W\phi'] \mid F\phi \mid F\{\chi\}\phi \mid G\phi \mid G\{\chi\}\phi\end{aligned}$$

( $Y$  is a propositional variable,  $\phi(Y)$  is syntactically monotone in  $Y$ )

$\mu$  and  $\nu$ : **recursion** (greatest fixed point and least fixed point)

$X, U, W, F, G$ : **action-based** versions of neXt, Until, Weak until, Future (“eventually”), Globally (“always”)

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## v-ACTL: variability-aware, action-based CTL

Additional **variability-aware** (action-based) versions of Box, Diamond, neXt, Future, Globally

$\langle \chi \rangle^{\square} \phi$  a next state exists, reachable by a **must** transition, executing an action satisfying  $\chi$ , in which  $\phi$  holds

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$X^{\square} \phi$  in the next state in the path, reached by a **must** transition  
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any formula that is true for MTS, is also true for all products (LTS)

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## Live states use SPL-specific information

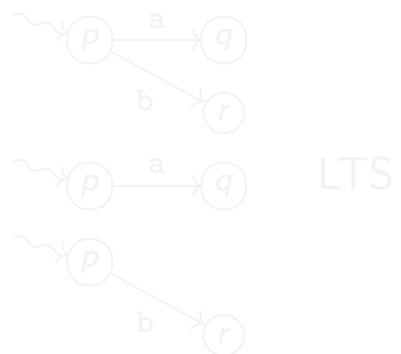
$$S \models \phi \Rightarrow S_p \models \phi \quad \forall \text{ product LTS } S_p \text{ of MTS } S$$

Recall: all (reachable) must transitions are preserved ( $\langle \rangle^\square, []^\square, X^\square, F^\square$ )

Live action sets define live states (not occur as final in any product)



Assume  
a OR b



In any product in which  $p$  occurs,  
 $p$  has at least one outgoing transition

$\Rightarrow p$  is a live state, since a OR b gives rise to a live action set  $\{a, b\}$

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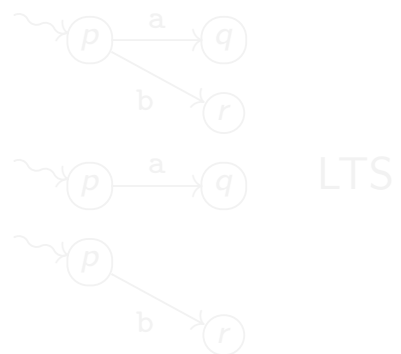
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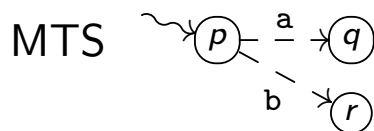
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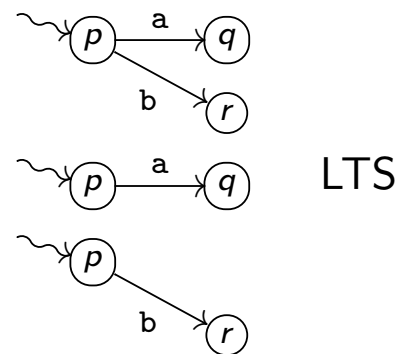
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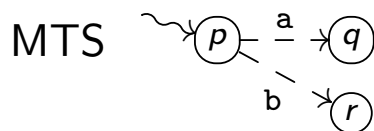
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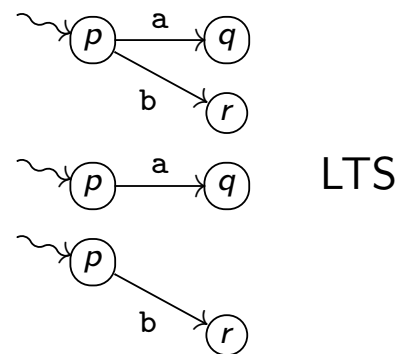
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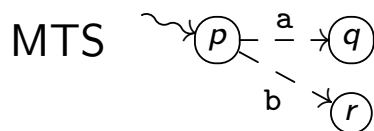
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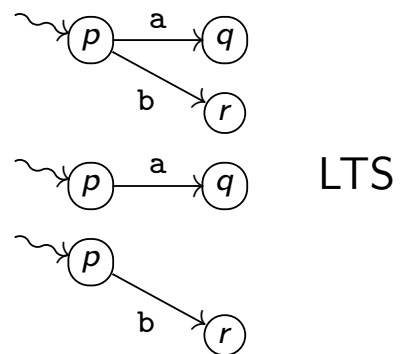
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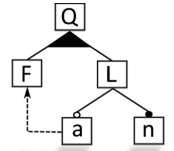


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 $p$  has at least one outgoing transition

$\Rightarrow p$  is a live state, since a OR b gives rise to a live action set  $\{a, b\}$

# QFLan: a framework for quantitative modelling and analysis of highly (re)configurable systems

ter Beek et al. @ IEEE TSE, 2018, FM'18



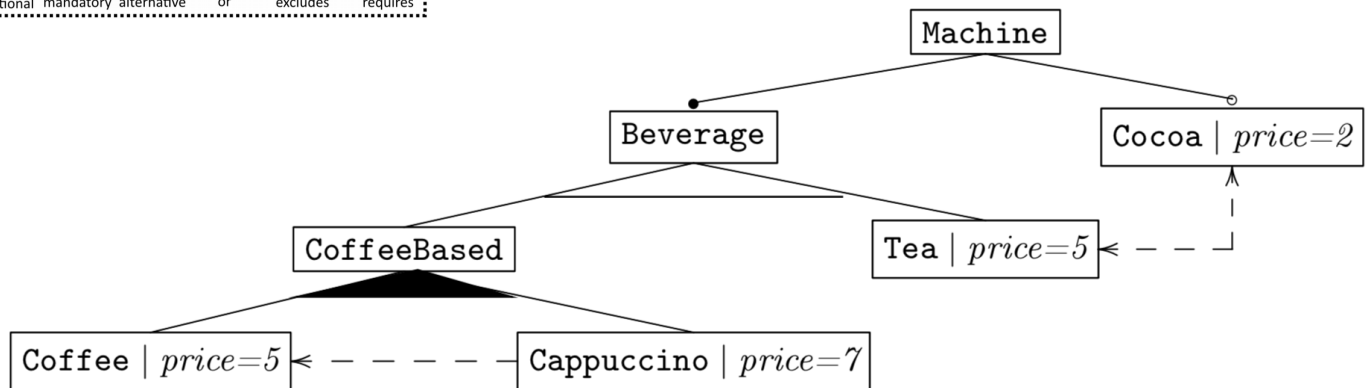
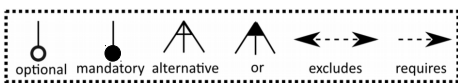
The screenshot displays the QFLan software interface. On the left is the **Project Explorer** showing a project structure with folders like 'benchmarks', 'fase2018', and 'VendingMachine.qflan'. Below it is the **Outline View** showing a hierarchical tree of the model's components. The central **QFLan Editor** shows the source code for a 'VendingMachine' model, including variable declarations, abstract and concrete features, and a feature diagram. Below the editor is the **Console View** displaying the output of the MultiVeStA client, including iteration counts and simulation run statistics. On the right is the **Plot View** showing a line graph of 'Means estimations' over time (x-axis from -1 to 101). The plot shows several data series (obs1, obs2, obs3, obs4, obs5) that stabilize around a mean value of approximately 8. The plot title is 'MultiVeStA analysis of VendingMachine.qflan SMC of queryVendingMachine.quatex. CI=(0.05,[0.5,0.05,0.05,0.05,0.05])'.

<https://github.com/qflanTeam/QFLan/>

*(more in his lectures!)*

These slides on QFLan are largely based on slides by Andrea Vandin

# A simple coffee vending machine product line: attributed feature model with quantitative constraints



```

begin abstract features
  Machine Beverage CoffeeBased
end abstract features

begin concrete features
  Cocoa Tea Cappuccino Coffee
end concrete features

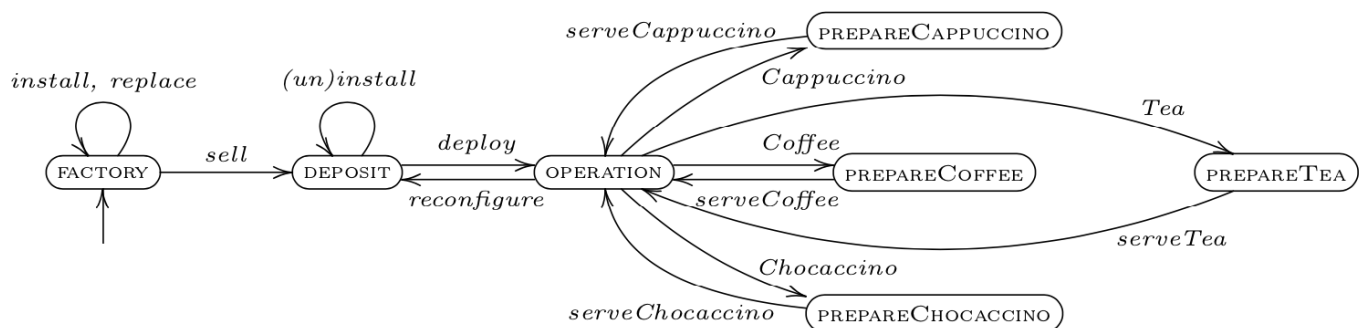
begin feature diagram
  Machine -> {?Cocoa, Beverage}
  Beverage -XOR-> {CoffeeBased, Tea}
  CoffeeBased -OR-> {Cappuccino, Coffee}
end feature diagram

begin feature predicates
  price= { Cappuccino = 7, Coffee = 5,
           Cocoa = 2, Tea = 5 }
end feature predicates

begin quantitative constraints
  { price(Machine) <= 10 }
end quantitative constraints

begin cross-tree constraints
  Cappuccino requires Coffee
  Tea excludes Cocoa
end cross-tree constraints
  
```

# A simple coffee vending machine product line: probabilistic behaviour with action constraints



```
begin variables
sold = 0
deploys = 0
end variables

begin actions
sell deploy reconfigure
chocaccino
serveCoffee serveCappuccino
serveChocaccino serveTea
end actions
```

```
begin action constraints
do(chocaccino) -> (has(Cappuccino)
and has(Cocoa))
end action constraints
```

```
begin init
installedFeatures = { Coffee }
initialProcesses = dynamics
end init
```

```
begin processes diagram
begin process dynamics
```

```
states = factory , deposit , operating , prepareCoffee ,
prepareCappuccino, prepareTea , prepareChocaccino
```

```
transitions =
//Factory
factory -(replace(Coffee,Tea),20)->factory,
factory -(install(Cocoa),10)->factory,
factory -(install(Cappuccino),10)->factory,
factory -(sell,1,{sold=1})-> deposit,

//Deposit
deposit -(install(Cappuccino),2.0)->deposit,
deposit -(uninstall(Cappuccino),2.0)->deposit,
deposit -(install(Cocoa),2.0)->deposit,
deposit -(uninstall(Cocoa),2.0)->deposit,
deposit -(deploy,2,{deploys=deploys+1})-> operating
```

```
//Operating
//Coffee
operating -(Coffee,3)-> prepareCoffee,
prepareCoffee -(serveCoffee,1) -> operating,
//Cappuccino
operating -(Cappuccino,3)-> prepareCappuccino,
prepareCappuccino -(serveCappuccino,1) -> operating,
//Chocaccino
operating -(chocaccino,2)-> prepareChocaccino,
prepareChocaccino -(serveChocaccino,1) -> operating,
//Tea
operating -(Tea,3)-> prepareTea,
prepareCappuccino -(serveTea,1) -> operating,
```

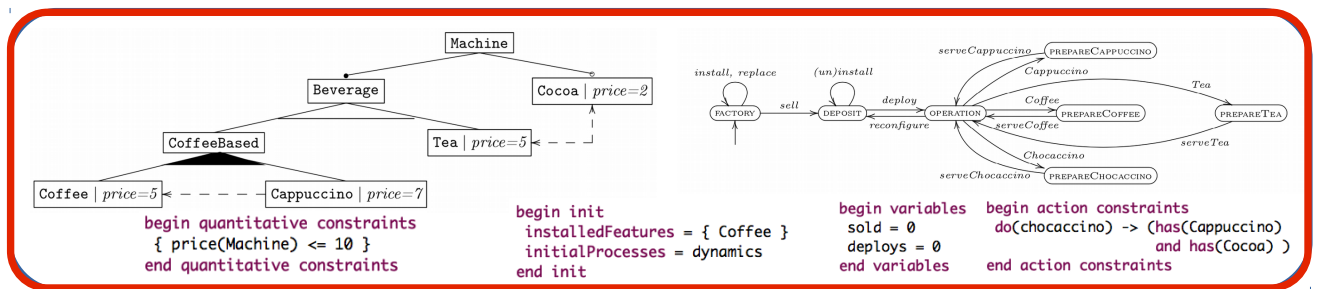
```
operating -(reconfigure,1) -> deposit
end process
end processes diagram
```

# A simple coffee vending machine product line: statistical model checking with MultiVeSta

Sebastio & Vandin @ VALUETOOLS'13

*(more in his lectures!)*

Pinpoint the precise moment in which a coffee machine is sold



begin analysis

```

query = eval when {sold == 1.0 } :
{ price(Machine) [delta=0.5],
  Coffee , Tea , Cappuccino , Cocoa
}
  
```

```

default delta=0.05
alpha = 0.05
parallelism = 1
  
```

end analysis

begin processes diagram  
begin process dynamics

```

states = factory , deposit , operating , prepareCoffee ,
  prepareCappuccino , prepareTea , prepareChocaccino
  
```

```

transitions =
//Factory
factory -(replace(Coffee,Tea),20)->factory,
factory -(install(Cocoa),10)->factory,
factory -(install(Cappuccino),10)->factory,
factory -(sell,1,{sold=1})-> deposit,
  
```

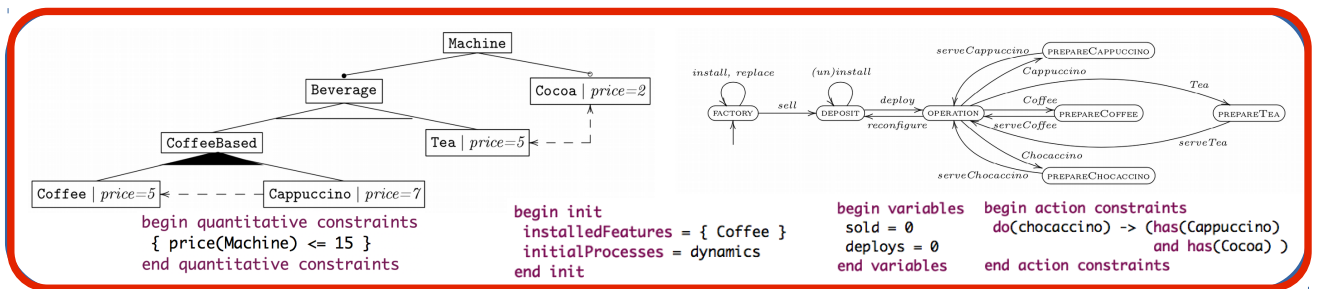
```

//Deposit
deposit -(install(Cappuccino),2.0)->deposit,
deposit -(uninstall(Cappuccino),2.0)->deposit,
deposit -(install(Cocoa),2.0)->deposit,
deposit -(uninstall(Cocoa),2.0)->deposit,
deposit -(deploy,2,{deploys=deploys+1})-> operating ,
  
```

# A simple coffee vending machine product line: statistical model checking with MultiVeSta

Sebastio & Vandin @ VALUETOOLS'13

Average price and probability of features of *(more in his lectures!)* sold coffee machines



begin analysis

```

query = eval when {sold == 1.0} :
{ price(Machine) [delta=0.5],
  Coffee , Tea , Cappuccino , Cocoa
}
  
```

```

default delta=0.05
alpha = 0.05
parallelism = 1
  
```

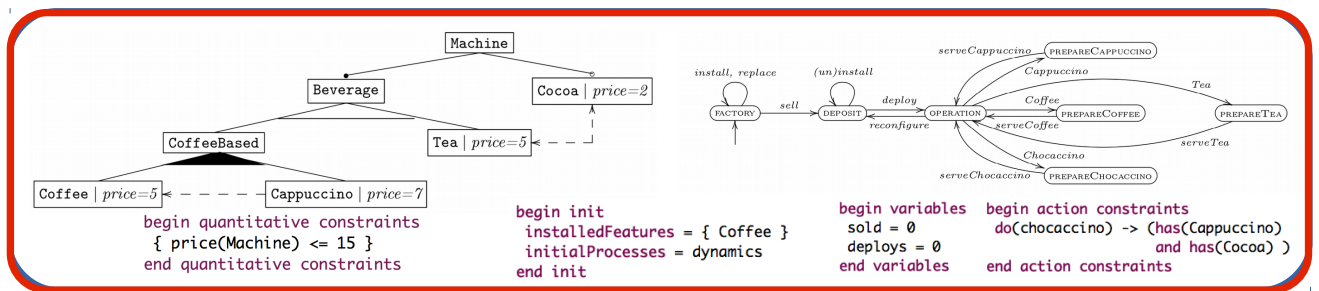
end analysis

	Price	Coffee	Tea	Cappuc cino	Cocoa
price(machine) <= 10	5.68	0.36	0.64	0.00	0.34
price(machine) <= 15	9.07	0.49	0.51	0.45	0.44

# A simple coffee vending machine product line: statistical model checking with MultiVeSta

Sebastio & Vandin @ VALUETOOLS'13

Average price and probability of features at *(more in his lectures!)* varying of time



begin analysis

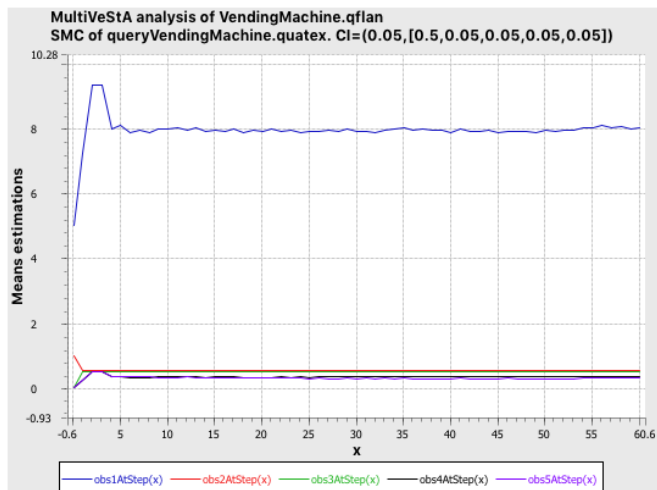
```

query = eval from 0 to 60 by 1 :
{ price(Machine) [delta=0.5],
  Coffee , Tea , Cappuccino , Cocoa
}
  
```

```

default delta=0.05
alpha = 0.05
parallelism = 1
  
```

end analysis



## Dedicated vs. off-the-shelf model checkers

Recall: dedicated SPL model checkers like SNIP, VMC, QFLan, etc.

 Dedicated model checkers need to be maintained and optimised

Dimovski *et al.*, Family-based model checking without a family-based model checker @ SPIN'15

Chrszon *et al.*, Family-based modeling and analysis for probabilistic systems – featuring ProFeat @ FASE'16

Dimovski *et al.*, Variability-specific abstraction refinement for family-based model checking @ FASE'17

Chrszon *et al.*, ProFeat: feature-oriented engineering for family-based probabilistic model checking @ FAC, 2018

Our approach towards family-based model checking with mCRL2:

- Product-based model checking of FTSs with mCRL2

ter Beek & de Vink @ FormaliSE'14, SPLC'14

- Branching feature bisimulation for FTSs

Belder, ter Beek & de Vink @ FMSPLE'15

- Feature-oriented modal  $\mu$ -calculi for reasoning on FTSs

ter Beek, de Vink & Willemse @ FMSPLE'16







- Family-based model checking of FTSs with mCRL2 as is

ter Beek, de Vink & Willemse @ FASE'17

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Belder, ter Beek & de Vink @ FMSPL'15

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ter Beek, de Vink & Willemse @ FMSPL'16

- **Family-based model checking** of FTSs with mCRL2 **as is**

ter Beek, de Vink & Willemse @ FASE'17

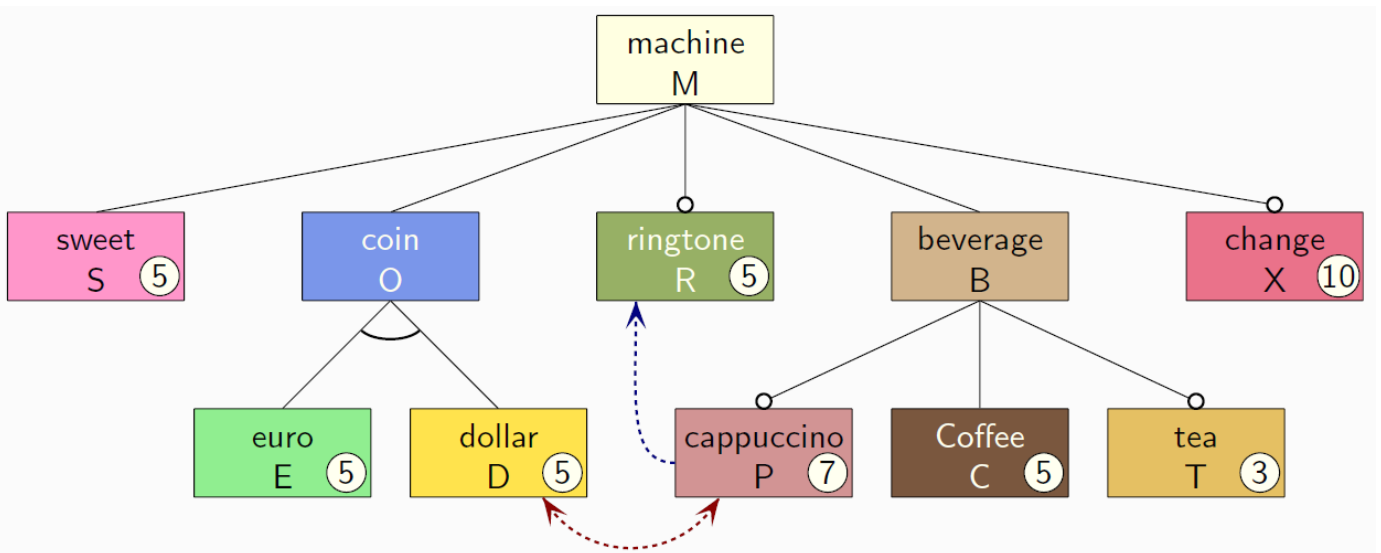
*ter Beek, van Loo, de Vink & Willemse @ FASE'20*

## Model checking the coffee machine with mCRL2

Recall: a product line or family of coffee machines

- *initially*, money must be inserted: either at least one euro's worth in coins, *exclusively* for European products, or at least one dollar's worth in coins, *exclusively* for Canadian products
- input of money can be cancelled via a cancel button; *optionally*, the coffee machine returns change if more than one euro or one dollar was inserted
- *once* the coffee machine contains at least one euro or one dollar, the user chooses whether (s)he wants sugar, by pressing one out of two buttons, *after* which (s)he can select a beverage
- the choice of beverage (coffee, tea, cappuccino) varies, but coffee *must* be offered by all products, whereas cappuccino *may* be offered *solely* by European products
- *optionally*, a ringtone *may* be rung *after* delivering a beverage; a ringtone *must* be rung by *all* products offering cappuccino
- *after* the beverage is taken, the coffee machine returns idle

## The coffee machine: attributed feature model



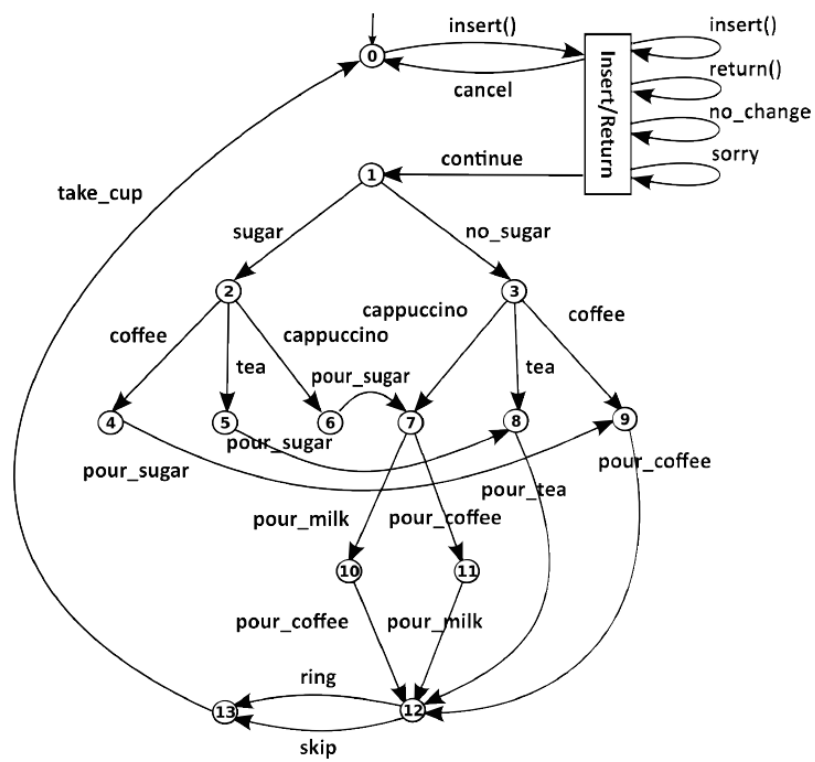
total cost  $\leq 30$

Recall:

Non-functional attributes:  $cost(product) = \sum \{ cost(feature) \mid feature \in product \}$

From  $2^{10} - 1 \xrightarrow{\text{feature diagram}} 2^5 \xrightarrow{\text{cross-tree constraints}} 20 \xrightarrow{\text{attributes}} 16$  valid products

# The coffee machine: family behaviour



FTS feature expression labels omitted for readability

## mCRL2 code: declarations

sort

```
Feature = struct M | S | O | R | B | X | D | E | P | T | C ;
FSet = List( Feature );
Coin = struct dime | quarter | half | dollar |
      ct10 | ct20 | ct50 | euro ;
Currency = struct Dollar | Euro ;
```

act

```
insert, return : Coin ;
continue, cancel, sorry, no_change,
sugar, no_sugar, coffee, tea, cappuccino,
pour_sugar, pour_milk, pour_coffee, pour_tea,
ring, skip, take_cup ;
setS, setO, setR, setB, setX,
setD, setE, setP, setT, setTP, setC ;
set_ko, ctc_ok ;
set_ok : FSet ;
cost : Int ;
```

## mCRL2 code: auxiliary operations (1/3)

```
map isSorted: FSet -> Bool;
  noDuplicates: FSet -> Bool;
  isSet: FSet -> Bool;
var ft,ft': Feature;
  fset: FSet;
eqn isSorted([]) = true;
  isSorted([ft]) = true;
  isSorted(ft |> (ft' |> fset)) =
    ft <= ft' && isSorted(ft' |> fset);
  noDuplicates([]) = true;
  noDuplicates(ft |> fset) =
    !(ft in fset) && noDuplicates(fset);
  isSet(fset) = isSorted(fset) && noDuplicates(fset);
```

## mCRL2 code: auxiliary operations (2/3)

```
map insert: Feature # FSet -> FSet;
var ft, ft': Feature;
    fset: FSet;
eqn insert(ft, []) = [ft];
    (ft < ft') -> insert(ft, ft' |> fset) = ft |> ft' |> fset;
    (ft == ft') -> insert(ft, ft' |> fset) = ft' |> fset;
    (ft > ft') -> insert(ft, ft' |> fset) = ft' |> insert(ft, fset);

map union: FSet # FSet -> FSet;
var ft, ft': Feature;
    fset, fset': FSet;
eqn union([], fset) = fset;
    union(fset, []) = fset;
    (ft < ft') -> union(ft |> fset, ft' |> fset') =
        ft |> union(fset, ft' |> fset');
    (ft == ft') -> union(ft |> fset, ft' |> fset') =
        ft' |> union(fset, fset');
    (ft > ft') -> union(ft |> fset, ft' |> fset') =
        ft' |> union(ft |> fset, fset');
```

## mCRL2 code: auxiliary operations (3/3)

```
map fcost : Feature -> Int ;
```

```
eqn fcost(M) = 0 ;
```

```
fcost(S) = 5 ;
```

```
fcost(O) = 0 ;
```

```
fcost(B) = 0 ;
```

```
fcost(R) = 5 ;
```

```
fcost(D) = 5 ;
```

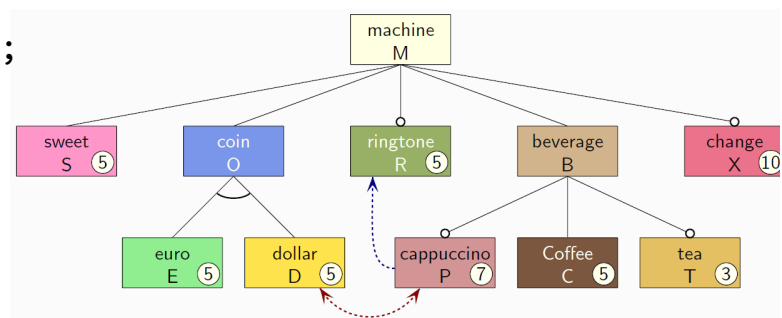
```
fcost(E) = 5 ;
```

```
fcost(X) = 10 ;
```

```
fcost(C) = 5 ;
```

```
fcost(T) = 3 ;
```

```
fcost(P) = 7 ;
```



```
map tcost : FSet -> Int ;
```

```
var ft : Feature ;
```

```
fset : FSet ;
```

```
eqn tcost([]) = 0;
```

```
tcost(ft |> fset) = fcost(ft) + tcost(fset) ;
```

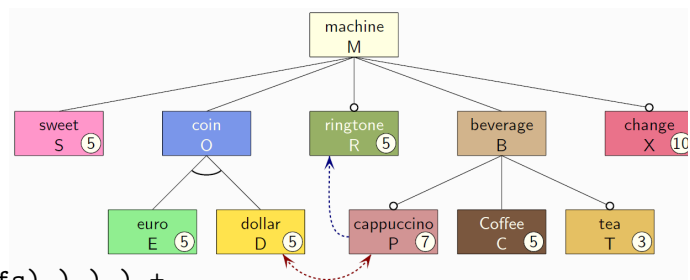
## mCRL2 code: selection process Sel (1/2)

```

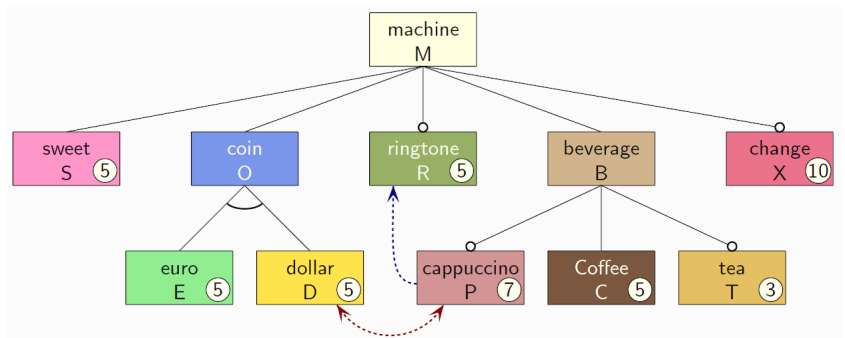
init
  Sel(0,[M]) ;

proc Sel(st:Int,fs:FSet) =
  %% feature states
  ( st == 0 ) -> (
    ( M in fs ) -> ( setS . Sel(1, insert(S,fs) ) ) ) +
  ( st == 1 ) -> (
    ( M in fs ) -> ( setO . Sel(2, insert(O,fs) ) ) ) +
  ( st == 2 ) -> (
    ( M in fs ) ->
      ( tau . Sel(3, fs ) + setR . Sel(3, insert(R,fs) ) ) ) +
  ( st == 3 ) -> (
    ( M in fs ) -> ( setB . Sel(4, insert(B,fs) ) ) ) +
  ( st == 4 ) -> (
    ( M in fs ) ->
      tau . Sel(5, fs ) + setX . Sel(5, insert(X,fs) ) ) +
  ( st == 5 ) -> (
    ( O in fs ) -> (
      setD . Sel(6, insert(D,fs) ) + setE . Sel(6, insert(E,fs) ) ) ) +
  ( st == 6 ) -> (
    ( B in fs ) -> (
      tau . Sel(7, fs ) + setT . Sel(7, insert(T,fs) ) +
      setP . Sel(7, insert(P,fs) ) + setTP . Sel(7, union([T,P],fs) ) ) ) +
  ...

```



## mCRL2 code: selection process Sel (2/2)



```

...
( st == 7 ) -> (
  ( B in fs ) -> ( setC . Sel(8, insert(C,fs) ) )
) +
%% cross-tree constraints
( st == 8 ) -> (
  ( ( D in fs ) && ( P in fs ) ) ->
    set_ko( fs ) . deadlock <>
  ( !( R in fs ) && ( P in fs ) ) ->
    set_ko( fs ) . deadlock <> ctc_ok . Sel(9,fs)
) +
%% attribute constraints
( st == 9 ) -> (
  ( tcost(fs) <= 30 ) ->
    set_ok( fs ) . cost( tcost(fs) ) . Prod(0,fs) <> set_ko . deadlock
) ;

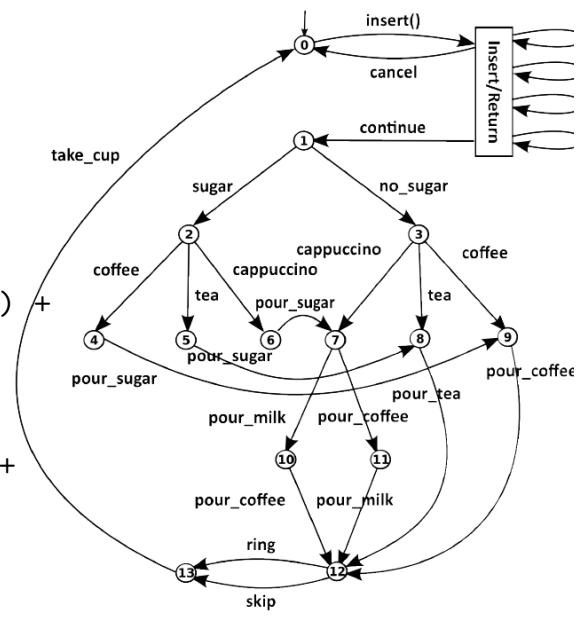
```

## mCRL2 code: product behaviour process Prod (1/2)

```

proc Prod(st:Int,fs:FSet) =
  ( st == 0 ) -> (
    Insert(0,fs) ) +
  ( st == 1 ) -> (
    ( S in fs ) -> ( sugar . Prod(2,fs) ) +
    ( S in fs ) -> ( no_sugar . Prod(3,fs) ) ) +
  ( st == 2 ) -> (
    ( C in fs ) -> coffee . Prod(4,fs) +
    ( T in fs ) -> tea . Prod(5,fs) +
    ( P in fs ) -> cappuccino . Prod(6,fs) ) +
  ( st == 3 ) -> (
    ( C in fs ) -> coffee . Prod(9,fs) +
    ( T in fs ) -> tea . Prod(8,fs) +
    ( P in fs ) -> cappuccino . Prod(7,fs) ) +
  ( st == 4 ) -> (
    ( M in fs ) -> ( pour_sugar . Prod(9,fs) ) ) +
  ( st == 5 ) -> (
    ( M in fs ) -> ( pour_sugar . Prod(8,fs) ) ) +
  ( st == 6 ) -> (
    ( M in fs ) -> ( pour_sugar . Prod(7,fs) ) ) +
  ...

```

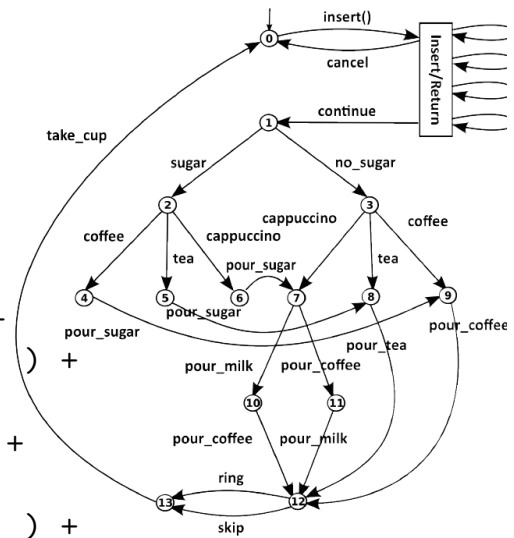


## mCRL2 code: product behaviour process Prod (2/2)

```

...
( st == 7 ) -> (
  ( M in fs ) -> ( pour_milk . Prod(10,fs) ) +
  ( M in fs ) -> ( pour_coffee . Prod(11,fs) ) ) +
( st == 8 ) -> (
  ( M in fs ) -> ( pour_tea . Prod(12,fs) ) ) +
( st == 9 ) -> (
  ( M in fs ) -> ( pour_coffee . Prod(12,fs) ) ) +
( st == 10 ) -> (
  ( M in fs ) -> ( pour_coffee . Prod(12,fs) ) ) +
( st == 11 ) -> (
  ( M in fs ) -> ( pour_milk . Prod(12,fs) ) ) +
( st == 12 ) -> (
  ( R in fs ) -> ( ring . Prod(13,fs) ) +
  ( !(R in fs) ) -> ( skip . Prod(13,fs) ) ) +
( st == 13 ) -> (
  ( M in fs ) -> ( take_cup . Prod(0,fs) ) ) ;

```

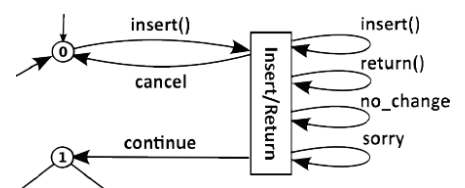


## mCRL2 code: money handling process Insert

```

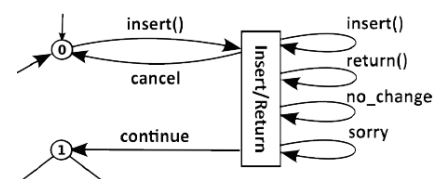
proc Insert(bal:Nat,fs:FSet) =
  ( bal < 100 ) -> (
    ( D in fs ) -> (
      insert(dime) . Insert(bal+10,fs) +
      insert(quarter) . Insert(bal+25,fs) +
      insert(half) . Insert(bal+50,fs) +
      insert(dollar) . Insert(bal+100,fs) ) +
    ( E in fs ) -> (
      insert(ct10) . Insert(bal+10,fs) +
      insert(ct20) . Insert(bal+20,fs) +
      insert(ct50) . Insert(bal+50,fs) +
      insert(euro) . Insert(bal+100,fs) ) ) +
  ( ( bal > 0 ) && ( bal < 100 ) ) ->
  Return(bal,fs) . cancel . Prod(0,fs) +
  ( bal >= 100 ) -> (
    ( ( !(X in fs) ) ->
      no_change . continue . Prod(1,fs) <>
      Return(Int2Nat(bal-100),fs) .
      continue . Prod(1,fs) )
  ) ;

```

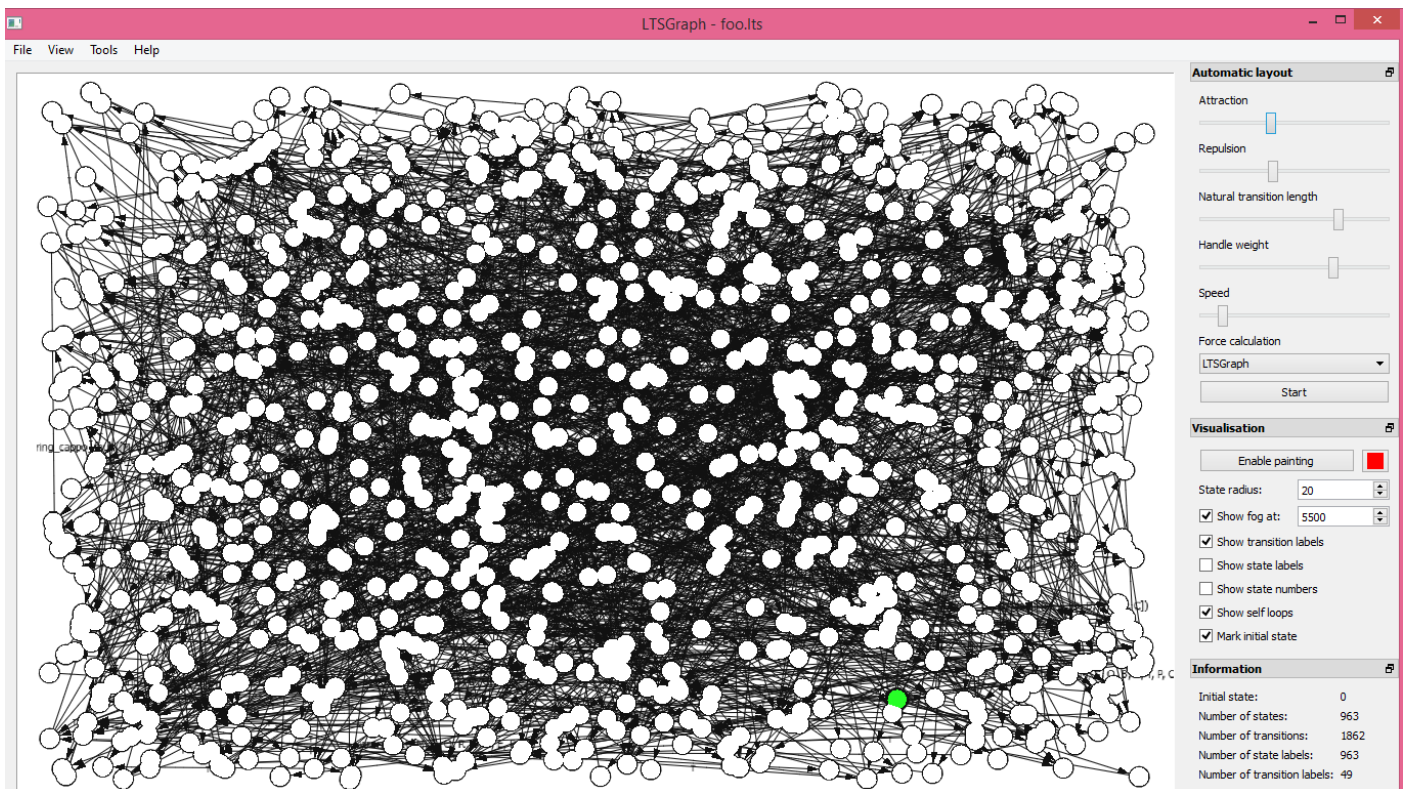


## mCRL2 code: money handling process Return

```
proc Return(bal:Nat,fs:FeatSet) =
  ( bal == 0 ) -> tau +
  ( D in fs ) -> (
    ( bal >= 50 ) ->
      return(half) . Return(Int2Nat(bal-50),fs) +
    ( ( bal < 50 ) && ( bal >= 25 ) ) ->
      return(quarter) . Return(Int2Nat(bal-25),fs) +
    ( ( bal < 25 ) && ( bal >= 10 ) ) ->
      return(dime) . Return(Int2Nat(bal-10),fs) +
    ( ( bal < 10 ) && ( bal > 0 ) ) ->
      sorry . Return(0,fs) ) +
  ( E in fs ) -> (
    ( bal >= 50 ) ->
      return(ct50) . Return(Int2Nat(bal-50),fs) +
    ( ( bal < 50 ) && ( bal >= 20 ) ) ->
      return(ct20) . Return(Int2Nat(bal-20),fs) +
    ( ( bal < 20 ) && ( bal > 0 ) ) ->
      return(ct10) . Return(Int2Nat(bal-10),fs) +
    ( ( bal < 10 ) && ( bal > 0 ) ) ->
      sorry . Return(0,fs)
  ) ;
```

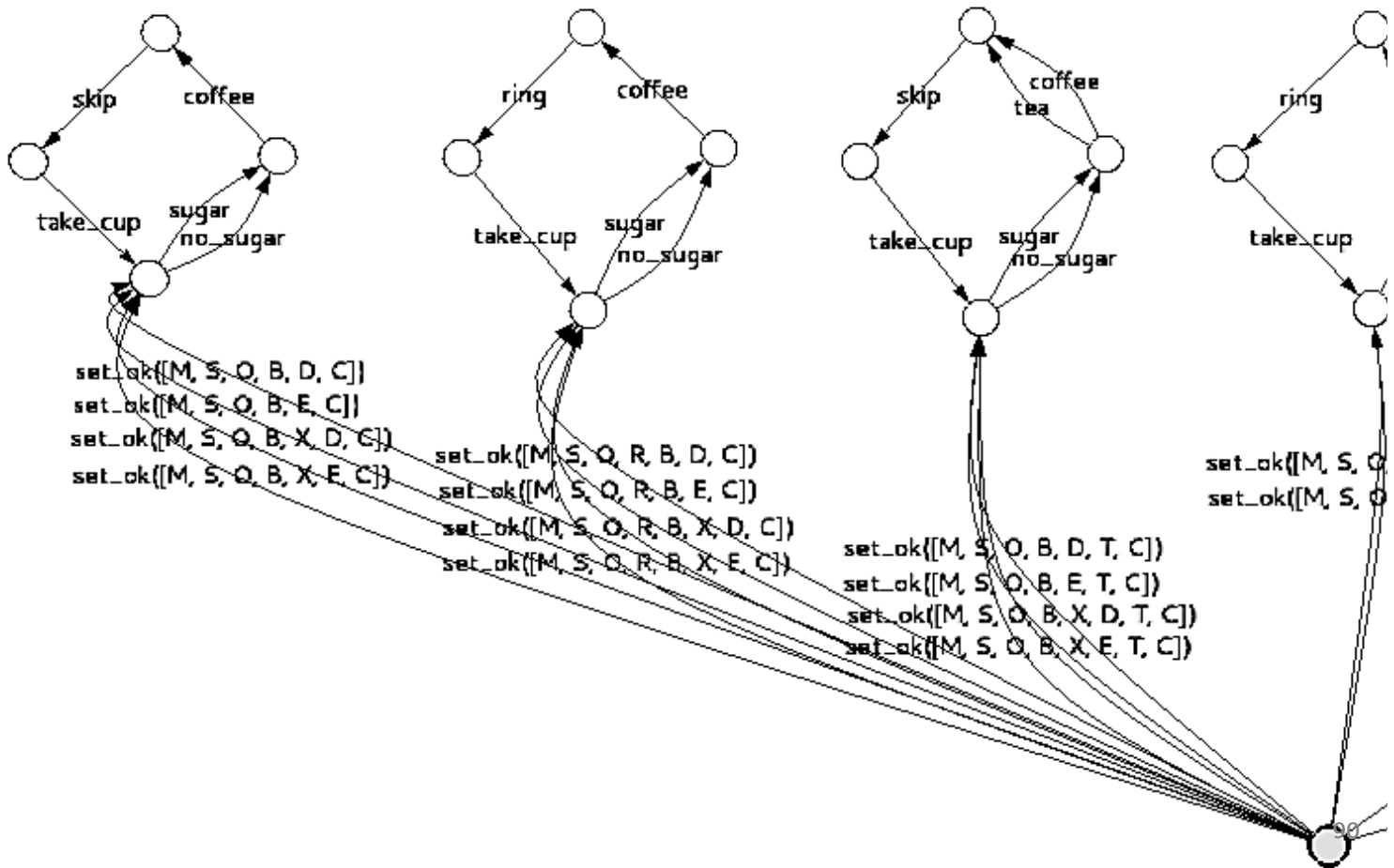


# Visual inspection of the state space

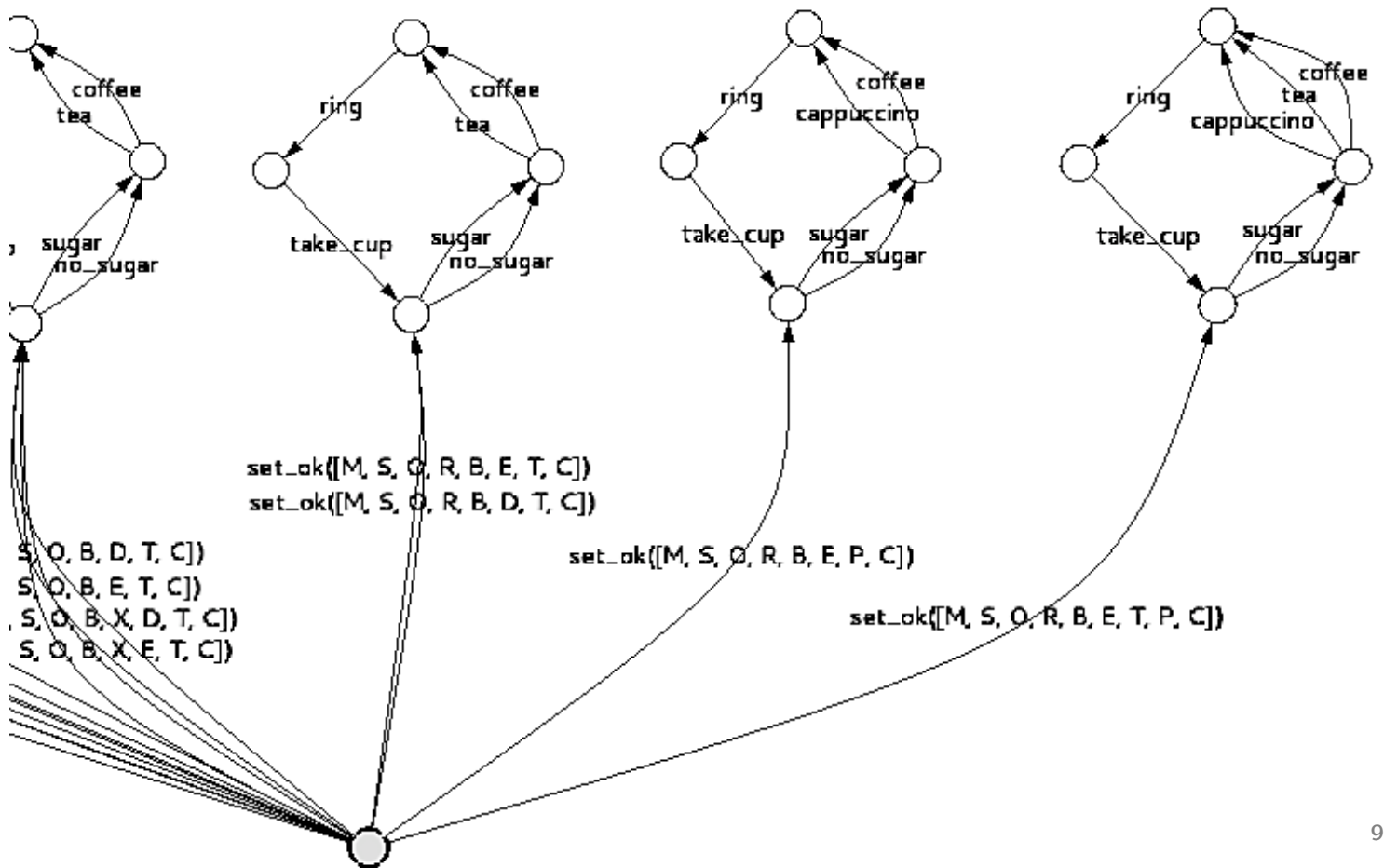




# Product behaviour abstracting from ...



## ... configuration and payment



## Product-based analysis with mCRL2: example properties

- `[(!continue)*.take_cup] false`  
if payment is not settled, no beverage is delivered
- `[true*.setX.true*.no_change] false`  
once feature `X` is selected, `no_change` will not occur
- `[true*] forall fs:FSet.`  
    `<set_ok(fs)> (val((D in fs) => !(P in fs)))`  
if a product is configured successfully,  
    then it cannot accept dollars and also provide cappuccino
- `mu Y.(<exists fs:FSet.set_ok(fs)> true ||`  
    `<exists fs:FSet.set_ko(fs)> true || [true] Y)`  
eventually either `set_ok` or `set_ko` can occur
- `forall c:Coin.[true*.insert(c)]`  
    `mu Y.(<cancel||take_cup> true || [true] Y)`  
after money is inserted, eventually a beverage is given  
    or money insertion is cancelled
- `[true*] forall n:Nat.[cost(n)](val(n<=30))`  
all valid products have a total cost at most 30

## Outlook

~~Next talk: Family Based Model Checking with mCRL2 (Erik de Vink)~~

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ter Beek & Legay @ QSPL special section, *LNCS FoMaC*, 2018

- Quantitative specification and verification techniques for systems with variability
- Modelling and analysis of real-time, hybrid or probabilistic systems with variability
- Analysis of safety, security or dependability properties of systems with variability
- Modelling and analysis of dynamic, adaptive and (runtime) reconfigurable systems
  - Variable UML Sequence Diagrams, Performance-Annotated UML Activity Diagrams
  - Featured Timed Automata, Configurable Parametric Timed Automata, Featured Modal Contract Automata, Featured Weighted Automata, Priced Featured Automata,
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