Team Behaviour in Eco-Grammar Systems *

Maurice H. ter Beek

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Grammar Systems

- Classic: one grammar, one language
- Grammar Systems: several grammars, one language

$\Rightarrow$ Cooperating Distributed

- sequential
- one sentential form

$\Rightarrow$ Parallel Communicating

- parallel
- each component its own sentential form
- alternate communicating and rewriting
- master

Eco-Grammar Systems

generative framework for describing ecosystems

- communities of evolving agents
- common evolving environment
- dynamic interaction

model life-like phenomena

- birth, death, pollution, hibernation, over-population, changes of season

Ecosystems

- environment (string)
- agents (strings)
- universal clock (rewriting step in each time unit)
- environment: evolution rules (L systems)
- agents: evolution rules (L systems) and action rules (*sequential* rewriting rules)

- (evolution) rules of environment independent of agents and environment

- evolution rules of agents *depend* on environment

- agents act on environment by action rules

- one in each time unit, depending on agent itself

- actions priority over evolution (environment)

- in time unit, only symbols unaffected by actions rewritten by evolution rules

Lindenmayer Systems

- biological motivation
- parallel rewriting (whole string)
- mathematical beauty

ET0L system $G = (\Sigma, \Delta, P, \omega)$

- Σ *total alphabet*
- $\Delta \subseteq \Sigma$ *terminal alphabet*
- P is a finite set of *tables*
- $\omega \in \Sigma^+$ *axiom*

Complete tables: $\forall \alpha \in V$ at least one production $\alpha \to \beta$, for $\beta \in V^*$.

ET0L system $G = (\Sigma, \Delta, P, \omega)$

- T0L system ($\Sigma = \Delta$)

- E0L system ($\#P = 1$)

- 0L system ($\Sigma = \Delta$ and $\#P = 1$)

- *Propagating* when every production is λ-free

Extended Tabled Simple Eco-Grammar System with Prescribed Teams

$$\Sigma = (E, R_1, R_2, \dots, R_n, Q_1, Q_2, \dots, Q_m),$$

- $E = (V_E, T_E, P_E, \omega)$,
  - V_E, the (finite) *environmental alphabet*,
  - $T_E \subseteq V_E$, the *terminal alphabet*,
  - P_E is a finite set of ET0L tables over V_E, the *evolution tables of the environment*,
  - $\omega \in V_E^+$, *initial state of the environment*,
- R_i is an *agent* which contains a finite set of context-free productions, the *action rules,*
- Q_j is a non-empty multiset of agents, a *prescribed team (of agents)*.

Only interested in environmental state, whole sentential form rewritten in each step.

Each rewriting step *uses* a team, i.e. if no team can be used the derivation is blocked.

Strong rewriting step ($\stackrel{s}{\Longrightarrow}_{Q_i}$): Apply in parallel 1 production from each agent of the team.

Weak rewriting step ($\stackrel{w}{\Longrightarrow}_{Q_i}$): Apply in parallel 1 production from each agent of the team *that has a production that can be applied.*

The parts of the sentential form that are not affected by the team, are rewritten by a table.

$$L_{-}(\Sigma) = \{z \in T_E^* \mid \omega \stackrel{-}{\Longrightarrow}_{Q_{i_1}} w_{i_1} \stackrel{-}{\Longrightarrow}_{Q_{i_2}} \cdots \stackrel{-}{\Longrightarrow}_{Q_{i_k}} w_{i_k} = z\}$$

PT_sETSEG_n / PT_wETSEG_n / $PT_xETSEG_n^\lambda$

$PT_xETSEG = \bigcup_{n\geq 1} PT_xETSEG_n$ / $x \in \{w, s\}$

$PT_sETSEG_0^\lambda = PT_wETSEG_0^\lambda = ET0L$

Examples

$$\Sigma_1 = ((\{a\},\{a\},\{a \to aa\},a),\{a \to aa\},\{a \to aa\})$$
$$L_s(\Sigma_1) = L_w(\Sigma_1) = \{a^{2^n} \mid n \geq 0\} \notin PR$$

$$\Sigma_2 = ((\{a\},\{a\},\{a \to aa\},a^4),\{a \to a\},\{a \to a\},$$
$$\{\{a \to a\},\{a \to a\}\})$$
$$L_s(\Sigma_2) = L_w(\Sigma_2) = \{a^2\}\{a^{2^n} \mid n \geq 1\} \notin T0L$$

$$\Sigma_3 = ((\{a,b,c\},\{a,b,c\},P_E,c^3),R_1,\ldots,R_6,$$
$$\{R_1,R_2,R_3\},\{R_4,R_5,R_6\}),$$

where

$P_E = \{\{a \to a, b \to b, c \to c\}\}$,

$R_1 = R_2 = R_3 = \{c \to ca\}$ and

$R_4 = R_5 = R_6 = \{c \to cb\}$

$$L_s(\Sigma_3) = L_w(\Sigma_3) = \{(cw)^3 \mid w \in \{a,b\}^*\} \notin E0L$$

$aaaa$	ccc
$aaaaaa$	$cacaca$
$aaaaaaaaaa$	$cbacbacba$
$\dots$	$\dots$
$\{a^2\}\{a^{2^n} \mid n \geq 1\}$	$\{(cw)^3 \mid w \in \{a,b\}^*\}$

Theorem 1 *For* $x \in \{s, w\}$

$$P\mathtt{0}L \subset PT_xSEG$$
$$EPOL \subset PT_xESEG$$
$$PT\mathtt{0}L \subset PT_xTSEG$$
$$EPT\mathtt{0}L \subset PT_xETSEG$$

All these and all coming results continue to hold when not restricted to λ-free productions.

Theorem 2 *For* $X \in \{E, T\}$

$$PR \subset PT_sXSEG \subseteq PT_sETSEG \subseteq PR_{ac},$$
$$PR_{ut} \subseteq PT_wXSEG \subseteq PT_wETSEG \subseteq PR_{ac},$$

$$PR^\lambda \subset PT_sXSEG^\lambda \subseteq PT_sETSEG^\lambda \subseteq RE \text{ and}$$
$$PR_{ut}^\lambda \subseteq PT_wXSEG^\lambda \subseteq PT_wETSEG^\lambda \subseteq RE.$$

Fernau '96: *If for some language family* $\mathcal{L}$ *we know that*

$$PR_{ut}^{[-\lambda]} \subseteq \mathcal{L} \subseteq PR_{ac}^{[-\lambda]}$$

and $\mathcal{L}$ *is closed under intersection with regular sets [and restricted homomorphisms], then* $\mathcal{L} = PR_{ac}^{[-\lambda]}$.

Normal Form for $PT_wETSEG^{(\lambda)}$

Single symbol axiom S (only in rules $S \to S$)

Unique *initial* team of one agent, containing only a production from the axiom to a set of nonterminals

Unique *terminal* team, containing only productions from nonterminals to terminals (or F)

Unique *terminal* table, containing only productions from nonterminals to terminals (or F)

In every table, a terminal is only replaced by F
In every non-terminal table, a nonterminal is only replaced by a string of nonterminals (or λ)

F is a distinguished nonterminal, only appearing as a left-hand side in rules $F \to F$

Lemma 1 *If Σ is an extended tabled simple eco-grammar system*

with prescribed teams,

operating in the weak rewriting step,

in normal form and

$H \in L(\Sigma)$,

then in every derivation of H in Σ only the first step uses the initial team (and no table) and only the last step uses the terminal team and the terminal table.

Lemma 2 *Both the families* PT_wETSEG *and* PT_wETSEG^λ *are closed under intersections with regular sets.*

Lemma 3 *The family* PT_wETSEG *is closed under restricted homomorphisms.*

Theorem 3

$$PT_wETSEG = PR_{ac} \text{ and}$$

$$PT_wETSEG^\lambda = RE.$$

Open Problems

Is weak rewriting more powerful than strong rewriting, or does $PT_sETSEG = PR_{ac}$ hold?

Does the relation between weak and strong rewriting apply to all subclasses as well or do different relations exist for different subclasses?

Note: also for other variants of grammar systems the relation between weak and strong rewriting is unknown.

Is $PR_{ut}^{[-\lambda]} \subseteq PR_{ac}^{[-\lambda]}$ strict?
In [Sto 71], the class of programmed grammars is claimed to be closed under intersection with regular sets (which would result in a proper inclusion indeed), but the proof is subject to disbelief ([Fer 96]).

Does the hierarchy that holds for L systems extend to the "L systems with teams"?