Simple eco-grammar systems with prescribed teams

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Extended tabled simple eco-grammar system with prescribed teams

$$\Sigma = (E, R_1, R_2, \ldots, R_n, Q_1, Q_2, \ldots, Q_m),$$

- $E = (V_E, T_E, P_E, \omega)$,
  - V_E, the (finite) *environmental alphabet*,
  - $T_E \subseteq V_E$, the *terminal alphabet*,
  - P_E is a finite set of ET0L tables over V_E, the *evolution tables of the environment*,
  - $\omega \in V_E^+$, *initial state of the environment*,
- R_i is an *agent* which contains a finite set of context-free productions, the *action rules,*
- Q_j is a finite set of agents, a *prescribed team (of agents)*.

In each step, the whole sentential form is rewritten.

Strong rewriting step ($\stackrel{s}{\Longrightarrow}_{Q_i}$): Apply in parallel a production from every agent of the team.

Weak rewriting step ($\stackrel{w}{\Longrightarrow}_{Q_i}$): Apply in parallel a production from every agent of the team *that has a production that can be applied.*

The parts of the sentential form that are not affected by the team, are rewritten by a table.

(Every rewriting step *uses* a team, i.e. if no team can be used the derivation comes to a halt.)

PT_sETSEG_n / PT_wETSEG_n

$PT_sETSEG = \bigcup_{n\geq 1} PT_sETSEG_n$

$PT_sETSEG_0^\lambda = PT_wETSEG_0^\lambda = ET0L$

The simple eco-grammar systems with prescribed teams

$$\Sigma_1 = ((\{a\}, \{a \rightarrow aa\}, a), \{a \rightarrow aa\}, \{a \rightarrow aa\}),$$

$$\Sigma_2 = ((\{a\}, \{a \rightarrow aa\}, a^4), \{a \rightarrow a\}, \{a \rightarrow a\}, \\ \{\{a \rightarrow a\}, \{a \rightarrow a\}\}) \text{ and}$$

$$\Sigma_3 = ((\{a, b, c\}, T, c^3), R_1, \ldots, R_6, \{R_1, R_2, R_3\}, \\ \{R_4, R_5, R_6\}),$$

where
$T = \{a \rightarrow a, b \rightarrow b, c \rightarrow c\}$,
$R_1 = R_2 = R_3 = \{c \rightarrow ca\}$ and
$R_4 = R_5 = R_6 = \{c \rightarrow cb\}$

generate the languages

$$L_s(\Sigma_1) = L_w(\Sigma_1) = \{a^{2^n} \mid n \geq 0\} \notin PR,$$

$$L_s(\Sigma_2) = L_w(\Sigma_2) = \{a^2\}\{a^{2^n} \mid n \geq 1\} \notin T0L \text{ and}$$

$$L_s(\Sigma_3) = L_w(\Sigma_3) = \{(cw)^3 \mid w \in \{a, b\}^*\} \notin E0L.$$

Lemma 1 *For* $X \in \{\lambda, E, T, ET\}$

$$PT_s XSEG = PT_{s,1} XSEG$$

Theorem 1 *For* $x \in \{s, w\}$

$$P0L \subset PT_x SEG$$
$$EP0L \subset PT_x ESEG$$
$$PT0L \subset PT_x TSEG$$
$$EPT0L \subset PT_x ETSEG$$

All these and all coming results continue to hold when not restricted to λ-free productions.

[m,LU,p] KVET0L system ([RvS 78])

$$G = (V, T, P, \omega)$$

P is a finite set of complete tables of the form $\{(\tau_1, c_1, r_1), (\tau_2, c_2, r_2), \ldots, (\tau_m, c_m, r_m)\}$, where

- τ_j is a context-free production,
- c_j is a multiset context condition and
- $r_j \in \mathbb{N}$ is a priority.

A production whose multiset *appears* in the sentential form can be applied if there is no production with a higher priority and a satisfied context condition.

$[m, LU, p]$ $KVET0L$

$[m, LU, p]$ $KVEPT0L = PR_{ac}$ ([RvS 78])

Lemma 2

$$PT_sETSEG \subseteq [m, LU, p]\ KVEPT0L$$

One rewriting step is simulated by two tables.

The first table is constructed for every team.

It simulates a team by replacing *one* occurrence of the left-hand side of one production of each agent of the team by its marked right-hand side *iff* all these left-hand sides occur in the sentential form (*multiset!*).

It copies all other symbols and marked symbols are replaced by a failure symbol.

The second table is constructed for every combination of a team and a table.

It simulates a table by using priorities:

(1) All marked symbols of the simulated team are replaced by a failure symbol *iff* they occur more than once in the sentential form.

(2) All marked symbols of the simulated team are replaced by their unmarked version *iff* they all occur once in the sentential form.

All unmarked symbols of the original alphabet are replaced by any of their right-hand sides that appeared in the table *iff* all marked symbols of the simulated team occur once in the sentential form.

(3) All marked symbols are replaced by a failure symbol.

Lemma 3

$$USC \subset PT_{s,1}ESEG$$
$$USC_{ut} \subseteq PT_{w,1}ESEG$$

For every scattered context rule

$$p : (\alpha_1, \alpha_2, \ldots, \alpha_s) \to (\beta_1, \beta_2, \ldots, \beta_s),$$

construct the team $Q_p = \{R_1, R_2, \ldots, R_s\}$ with

$$R_j = \{\alpha_j \to \beta_j\}, \text{ for } 1 \leq j \leq s.$$

Corollary 1

$$PUSC \subseteq PT_{s,1}SEG$$
$$PUSC_{ut} \subseteq PT_{w,1}SEG$$

Lemma 4

$$PT_wETSEG \subseteq PR_{ac}$$

Theorem 2 *For* $X \in \{E, T\}$

$$PR \subset PT_sXSEG \subseteq PT_sETSEG \subseteq PR_{ac},$$
$$PR_{ut} \subseteq PT_wXSEG \subseteq PT_wETSEG \subseteq PR_{ac},$$

$PR^{\lambda} \subset PT_sXSEG^{\lambda} \subseteq PT_sETSEG^{\lambda} \subseteq RE$ *and*
$PR^{\lambda}_{ut} \subseteq PT_wXSEG^{\lambda} \subseteq PT_wETSEG^{\lambda} \subseteq RE.$

Normal form for $PT_wETSEG^{(\lambda)}$

Single symbol axiom S (only in copying rules)

Unique *initial* team of one agent, containing only a production from the axiom to a set of nonterminals

Unique *terminal* team, containing only productions from nonterminals to terminals (or F)

Unique *terminal* table, containing only productions from nonterminals to terminals (or F)

In every table, a terminal is only replaced by F
In every non-terminal table, a nonterminal is only replaced by a set of nonterminals (or λ)

F is a distinguished nonterminal, only appearing in copying rules in all non-terminal teams and tables.

Lemma 5 *Families* PT_wETSEG *and* PT_wETSEG^λ *are closed under intersections with regular sets.*

Lemma 6 *The family* PT_wETSEG *is closed under restricted homomorphisms.*

Corollary 2.11 ([Fer 95]) If for some language family $\mathcal{L}$ *we know that*

$$\mathcal{L}(P, CF[-\lambda], ut) \subseteq \mathcal{L} \subseteq \mathcal{L}(P, CF[-\lambda], ac),$$

and $\mathcal{L}$ *is closed under intersection with regular sets [and restricted homomorphisms], then* $\mathcal{L} = \mathcal{L}(P, CF[-\lambda], ac)$. □

Theorem 3

$$PT_wETSEG = PR_{ac} \text{ and}$$

$$PT_wETSEG^\lambda = RE.$$

Open problems and future research

Is weak rewriting more powerful than strong rewriting, or does $PT_sETSEG = PR_{ac}$ hold?

Does the relation between weak and strong rewriting apply to all subclasses as well or do different relations exist for different subclasses?

Note: also for colonies the relation between weak and strong rewriting is unknown.

Can the conjecture that the inclusion between programmed grammars with unconditional transfer and those with appearance checking is proper be affirmed? In [Sto 71], the class of programmed grammars is claimed to be closed under intersection with regular sets (which would result in a proper inclusion indeed), but the proof is subject to disbelief ([Fer 95]).

Does the hierarchy that holds for L systems extend to the "L systems with teams"?

CD grammar systems with prescribed teams

Theorem 4 *For* $x \in \{s, *\}$, $f \in \{=1, \geq 1, *\} \cup \{\leq k \mid k \geq 1\}$ *and* $g \in \{*\} \cup \{\leq k, =k, \geq k \mid k \geq 1\}$

$$PR_{ut} = PT_w CD_1(f) \nsubseteq PT_x CD(g) \text{ and}$$
$$PR_{ut}^{\lambda} = PT_w CD_1^{\lambda}(f) \nsubseteq PT_x CD^{\lambda}(g).$$

Corollary 2 *For* $f \in \{=1, \geq 1, *\} \cup \{\leq k \mid k \geq 1\}$

$$CD(t) \subset PT_w CD_1(f) \subset PT_w CD_1^{\lambda}(f).$$