Teams in grammar systems

Maurice H. ter Beek

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History and motivation

Cooperating grammar systems were already introduced in [MR 78], but received more attention when the link with the **blackboard model of problem solving** from AI was established in [CK 89].

Knowledge sources (grammars) contribute to solving the problem by changing (rewriting) the current state of the blackboard (sentential form) under some control (derivation modes) until a solution (terminal word) is obtained.

Knowledge is **distributed** and **cooperation** takes place via the blackboard. The systematic study of grammar systems was started in [CD 90], where the name cooperating distributed (**CD**) **grammar systems** was introduced. The monograph [CDKP 94] contains an exhaustive overview of the **theory of grammar systems** until ca. 1992.

(Hybrid) CD grammar systems - definitions

In a CD grammar system

$$\Gamma = (N, T, S, P_1, P_2, \ldots, P_n)$$

(1) Rewriting rules are grouped in components

(2) **One** mode of derivation for the grammar system

(3) A rewriting step consists of choosing a component and rewrite the sentential form according to the mode of derivation, picking one rule from the component every time.

In a **hybrid** CD grammar system

$$\Gamma = (N, T, S, (P_1, f_1), (P_2, f_2), \ldots, (P_n, f_n))$$

(1), (3) and

(4) Every component can have a **different** mode of derivation

The modes of derivation are, for a $k \geq 1$

($\leq k$) **At most** k direct derivation steps in succesion by a component

($=k$) **Exactly** k direct derivation steps in succesion by a component

($\geq k$) **At least** k direct derivation steps in succesion by a component

($*$) **An arbitrary number** of direct derivation steps in succesion by a component

(t) **Maximal** rewriting, the component must rewrite as long as it can

(Hybrid) CD grammar systems - generative power

Teams in grammar systems - definitions

A **(prescribed) team CD grammar system** with teams of **variable** size ([PR 94]) is a construct

$$\Gamma = (N, T, S, P_1, P_2, \ldots, P_n, Q_1, Q_2, \ldots, Q_m),$$

where

- N is the set of **nonterminals**
- T is the set of **terminals**
- S is the **start symbol**
- P_i are sets of rewriting rules, called **components**
- $Q_i = \{P_{i_1}, P_{i_2}, \ldots, P_{i_{s_i}}\}$ are sets of components, called **teams**

A rewriting step of a team now consists of choosing exactly one rule from each of its components and rewrite the sentential form **in parallel** according to the mode of derivation.

A **(prescribed) team CD grammar system** with teams of **constant** size ([KMPS 93]) is a construct

$$\Gamma = (N, T, w, P_1, P_2, \ldots, P_n, Q_1, Q_2, \ldots, Q_m).$$

The **modes of derivation for teams** in grammar systems are

$\leq k$, $= k$, $\geq k$, $*$ and

(t_0) A team must rewrite until it can no longer rewrite as a team ([FP 95])

(t_1) A team must rewrite until no component can rewrite any longer ([KMPS 93])

(t_2) A team must rewrite until there is a component that cannot rewrite any longer ([PR 94])

An example of a team CD grammar system with teams of constant size

$$\Gamma_1 = (\{A, B\}, \{a, b, c\}, AB, P_1, P_2, P_3, P_4),$$

where

$$\begin{aligned} P_1 &= \{A \to a^4 A' b^4, A \to ab\}, \\ P_2 &= \{B \to c^4 B', B \to c\}, \\ P_3 &= \{A' \to A\} \text{ and} \\ P_4 &= \{B' \to B\}. \end{aligned}$$

Consider the following two teams of constant size, namely 2:

$$\begin{aligned} Q_1 &= \{P_1, P_2\} \text{ and} \\ Q_2 &= \{P_3, P_4\}. \end{aligned}$$

$$L_f(\Gamma_1, 2) = \{a^n b^n c^n | n = 1 \cup n \; mod \; 4 \equiv 1, n \geq 1\}$$

for $f \in \{=1, \geq 1, *, t_0, t_1, t_2\} \cup \{\leq k \mid k \geq 1\}$.

An example of a prescribed team CD grammar system with teams of variable size

$$\Gamma_2 = (\{S, A, B, C, D\}, \{a, b, c, d\}, S,$$
$$P_1, P_2, P_3, P_4, P_5, Q_1, Q_2, Q_3),$$

where

$$\begin{aligned}
P_1 &= \{S \to ABCD\}, \\
P_2 &= \{A \to aA, A \to a\}, \\
P_3 &= \{B \to bB, B \to b\}, \\
P_4 &= \{C \to cC, C \to c\}, \\
P_5 &= \{D \to dD, D \to d\}, \\
Q_1 &= \{P_1\}, \\
Q_2 &= \{P_2, P_4\} \text{ and} \\
Q_3 &= \{P_3, P_5\}.
\end{aligned}$$

$$L_f(\Gamma_2) = \{a^n b^m c^n d^m \mid m, n \geq 1\}$$

for $f \in \{=1, \geq 1, *, t_0, t_1, t_2\} \cup \{\leq k \mid k \geq 1\}$.

Hybrid teams in grammar systems - definitions

An **automatically formed hybrid team CD grammar system**

$$\Gamma = (N, T, S, (P_1, f_1), (P_2, f_2), \ldots, (P_n, f_n), (Q_1, g_1), (Q_2, g_2), \ldots, (Q_m, g_m))$$

is automatically constructed from a hybrid CD grammar system by forming teams such that

$$(Q_i, g_i) = \{(P_k, f_k) \mid f_k = g_i, (P_k, f_k) \in \{(P_1, f_1), (P_2, f_2), \ldots, (P_n, f_n)\}, 1 \le i \le m, 1 \le k \le n\}$$

A **prescribed hybrid team CD grammar system** is a construct

$$\Gamma = (N, T, S, P_1, P_2, \ldots, P_n, (Q_1, f_1), (Q_2, f_2), \ldots, (Q_m, f_m))$$

$\Rightarrow$ **Now different teams can have different modes of derivation**

An example of an automatically formed hybrid team CD grammar system with teams of variable size

Consider the hybrid CD grammar system

$$\begin{aligned}\Gamma_3 = (&\{S, A, B\}, \{a, b, c\}, S, (P_1, \leq 1),\\ &(P_2, =4), (P_3, =4), (P_4, =1), (P_5, =1)),\end{aligned}$$

where

$$\begin{aligned} P_1 &= \{S \to AB\},\\ P_2 &= \{A \to aAb\},\\ P_3 &= \{B \to cB\},\\ P_4 &= \{A \to ab\} \text{ and}\\ P_5 &= \{B \to c\}.\end{aligned}$$

This leads automatically to the following teams

$$\begin{aligned} (Q_1, \leq 1) &= \{(P_1, \leq 1)\},\\ (Q_2, =4) &= \{(P_2, =4), (P_3, =4)\} \text{ and}\\ (Q_3, =1) &= \{(P_4, =1), (P_5, =1)\}.\end{aligned}$$

$$L(\Gamma_3) = \{a^n b^n c^n \mid n = 1 \cup n \ mod \ 4 \equiv 1, n \geq 1\}.$$

An example of a prescribed hybrid team CD grammar system with teams of variable size

$$\Gamma_4 = (\{S, A, B, C, D\}, \{a, b, c, d\}, S, P_1, \ldots, P_6,$$
$$(Q_1, =1), (Q_2, =1), (Q_3, =1), (Q_4, =4)),$$

where

$$\begin{aligned}
P_1 &= \{S \to ABCD\}, \\
P_2 &= \{A \to aA\}, \\
P_3 &= \{B \to bB\}, \\
P_4 &= \{C \to cC\}, \\
P_5 &= \{D \to dD\}, \\
P_6 &= \{A \to a, B \to b, C \to c, D \to d\}, \\
Q_1 &= \{P_1\}, \\
Q_2 &= \{P_2, P_4\}, \\
Q_3 &= \{P_3, P_5\} \text{ and} \\
Q_4 &= \{P_6\}.
\end{aligned}$$

$$L(\Gamma_4) = \{a^n b^m c^n d^m \mid m, n \geq 1\}.$$

(Hybrid) teams in CD grammar systems - generative power

Sketch of the proof of a known result

Lemma 1 *([PR 94]) For $s \geq 2$*

$$PR_{ac} \subseteq T_sCD(t_1) \text{ and } PR_{ac}^{\lambda} \subseteq T_sCD^{\lambda}(t_1).$$

- A **string axiom** contains the original axiom and control symbols
- The **control symbols** guide the derivation
- For every rule in P, construct **five** 2-teams, divided in two parts
    1. choose a nonterminal to rewrite, rewrite it and hand over control to a rule from the **success field**
    2. If a nonterminal is not in the sentential form, test this and if true hand over control to a rule from the **failure field**
- **One more** team changes the control symbols to terminals iff the string is terminal
- **Trap rules** are used to prohibit the forming of 'unwanted' teams
- $T_sCD(t_1)$ is a **full AFL** ([KMPS 93])

Sketch of the proof of a new result

Lemma 2 *For* $s \geq 2$

$$HPT_*CD \subseteq MAT_{ac} \text{ and } HPT_*CD^\lambda \subseteq MAT_{ac}^\lambda.$$

- **Control symbols** guide the derivation
- A matrix contains **one rule from every component** of the team being simulated
- **Priming** guarantees rewriting of only those nonterminals that were already present in the original sentential form
- Modes $\leq k$, $= k$ and $\geq k$ require a **counter**
- Mode $*$ requires no further restriction

- Modes t_0, t_1 and t_2 require **appearance checking** to check the absence of any nonterminal that is present in the sentential form

(t_0) in **every possible sequence** of one rule from each component of the team being simulated

(t_1) in **every** component of the team being simulated

(t_2) in **at least one** component of the team being simulated

- Remove primes, start simulating a new team

- Replace control symbol by a terminal z

- MAT_{ac} is **closed under restricted morphisms**

Some open problems concerning teams

Open problem 1 *Do the inclusions* $PR_{ac}^{\lambda} \subseteq HT_*CD^{\lambda}$ *and* $PR_{ac} \subseteq HT_*CD$ *hold?*

Open problem 2 *Is the inclusion* $HCD \subseteq HT_*CD$ *a proper one?*

Remark 1 *A proper inclusion of the above open problem would result in the proper inclusion* $HCD \subset MAT_{ac}$, *an open problem stated in [Păun 94].*

Open problem 3 *What is the generative power of prescribed (hybrid) team CD grammar systems with only regular or linear rules and teams of constant size* s, *for* $s \geq 2$?

Before stating one more open problem, a new result is presented in the following lemma.

Lemma 3 *For* $X \in \{MLIN, CF\}$ *and* $f \in \{=1, \geq 1, *\} \cup \{\leq k \mid k \geq 1\}$

$$PT_*CD_X(f) = PT_*CD_{X,1}(f) \text{ and } PT_*CD_X^\lambda(f) = PT_*CD_{X,1}^\lambda(f).$$

Open problem 4 *What is the generative power of (hybrid) prescribed team CD grammar systems with teams of variable size and at most* p*,* $p \geq 2$*, productions per component, for different types of rules and modes of derivation?*

Controlled grammar systems - definitions

Three types of control mechanisms are considered to be added to the prescribed hybrid team CD grammar systems

1. External control by a **graph**
2. Internal control by a **hypothesis language**
3. Intermediate control by a **gsm**

These ideas are not new, to name but a few

[MR 78] Cooperating grammar systems with (1)

[CD 90] CD grammar systems with (1)

[Friš 68] Context-free grammars with (2)

[Das 91] CD grammar systems with (2)

[Pāun 95] Colonies with (2) or with (3)

[Mit 90] CD grammar systems with (3)

An example of a hybrid CD grammar system with graph control

$$\Gamma_5 = (\{S, A, B\}, \{a, b\}, S, (P_1, t), (P_2, {=}1), (P_3, t), (P_4, t), (P_5, t), U),$$

where

$$\begin{aligned} P_1 &= \{S \to ABS, S \to AB\}, \\ P_2 &= \{A \to a\}, \\ P_3 &= \{B \to bB'\}, \\ P_4 &= \{B \to b\}, \\ P_5 &= \{B' \to B\} \text{ and} \\ U &\text{ is the following graph.} \end{aligned}$$

$$\begin{array}{ccccc} P_1 & \longrightarrow & P_2 & \longrightarrow & P_4 \\ & \nearrow & \downarrow & & \\ P_5 & \longleftarrow & P_3 & & \end{array}$$

$$L^U(\Gamma_5) = \{(ab^n)^n \mid n \geq 1\}.$$

Future research - is there life beyond teams?